Logical Foundations *of* Induction

[*al-Usus al-Mantiqiyyah li*’*l-Istiqra*’]

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THE OBJECT OF OUR STUDY IS TWO FOLD: First, we are concerned to show the logical foundations of inductive inference which embraces all scientific inferences based on observation and experiment. In this context we have offered a new explanation of human knowledge based on inductive inference.

Secondly, we are interested to show certain conclusions connected with religious beliefs based on our study of induction. That is, the logical foundations of all scientific inferences based on observation and experiment are themselves the logical foundations on which a proof of the existence of God can be based. This proof is a version of the argument from design, and is inductive in its character.

Now, we have to choose the whole scientific knowledge or reject it, and then an inductive proof of the existence of God would be on the same footing as any scientific inference. Thus, we have found that science and religion are connected and consistent, having the same logical basis; and cannot be divorced. Such logical connection between the methods of science and the method of proving God's existence may be regarded as the ground of understanding the divine direction, in the Qur'an, the Holy Book of Muslims, to observe the workings of the natural world.

The Qur'an encourages scientific knowledge on empirical grounds, and in this sense, the argument from design is preferred in the Qur'an to other proofs of the existence of God, being akin to sense and concreteness and far from abstractions and sheer speculations.

Part I

Induction *and* Epistemology

CHAPTER I

Aristotelian Induction

Meanings of Induction

Induction is a sort of inference proceeding from particular proposition to general ones; the former being based on observation and experiment. Observation is meant as one's attention to a certain natural phenomenon as actually occurring, to discover its causes and relations to other phenomena. By experiment is meant one's interference and effort to produce such a phenomenon in a variety of circumstances, to discover those causes and relations. The difference between observation and experiment is that between observing lightning, for instance, as it naturally occurs, and actively producing it in a certain way in the laboratory. Thus, inductive inference begins with observing a certain phenomenon or actively producing it in many cases, and then establishing a general conclusion suggested by these observations and experiments.

Aristotle did not distinguish between observation and experiment, and considered induction as any inference based on enumerating particular instances, consequently, he classified induction into perfect and imperfect, if the conclusion refers to all the particulars in question, induction is perfect, if it includes reference to some particular instances only, induction is imperfect [1].

Aristotle has considered perfect induction in a way different from his consideration of imperfect induction. Induction cannot be divided, in our view, into perfect and imperfect because induction in fact proceeds from particular to universal, whereas perfect induction does not do so, but its premises are general like its conclusion. Thus, we regard perfect induction as deduction not induction; and it is imperfect induction that is induction proper.

Aristotle's perfect induction

Perfect induction was of great logical value for Aristotle being as rigorous as syllogism. When syllogism predicates major terms of minor term by virtue of a middle term, its conclusion is certain; similarly, the conclusion of perfect induction relates a predicate to a subject by means of enumerating all instances of that subject, thus the certainty of such conclusion. Further, Aristotle considers perfect induction a basis of recognizing the ultimate premises of syllogistic reasoning.

We reach those premises not by syllogism but by perfect induction. For, in syllogism we predicate the major term to the minor term by means of the middle term, this being a predicate of the minor term and subject of the major term; and if we try to prove syllogistically that the major term is asserted of the middle term, or that the middle term is asserted of the minor term, we have to find out the middle term between them, and then we go on until we reach ultimate premises wherein we relate predicate to subject without any medium.

And as we cannot get a syllogism without a middle term, the only way for Aristotle to reach such ultimate premises of syllogism is by perfect induction. Later on, medieval logicians did not give such a great value to perfect induction, but they still regarded it as an important means of arriving at ultimate premises.

Criticism of perfect induction

Our comments on Aristotelian perfect induction are as follows:

(1) We are concerned in this book with induction proceeding from particular to universal, thus perfect induction lies outside our interest, since it is a sort of deduction the premises of which are also universal, and the principle of non contradiction is sufficient to show the truth of its conclusion.

(2) We may ask, what is the use of the conclusion of perfect induction for us? Two Aristotelian answers are expected, (i) the conclusion asserts a logical or causal relation between its two terms. When we say John, Peter and Smith are all the individuals of the human species; John, Peter and Smith eat; therefore every man eats. It may here be said that the conclusion asserts a causal relation between humanity and eating, (ii) Aristotle may not insist on regarding the conclusion as asserting a causal relation, but show the fact that men eat, by complete enumeration of all individuals.

Let us discuss these answers. Aristotle would be mistaken if he thought that perfect induction gives a causal relation between the terms of the conclusion otherwise this conclusion would give new information not included in the premises; and then the inductive reasoning loses its logical validity and cannot be explained by the law of non - contradiction alone. Further, if we take the conclusion of perfect induction as giving a fact about its terms and not a certain relation between them, such a conclusion would indeed be valid since it is contained in the premise, but then perfect induction would not be a proof in Aristotle's sense. He conceived proof as giving a logically certain relation between the terms of the conclusion, and this certainty arises from our discovery of the true cause of that relation. Such a cause may be the subject itself and the predicate may be either an essential attribute or not; if essential attribute, then the conclusion is an ultimate premise, but if not, the conclusion would be demonstrated only in a secondary sense.

Now, if the conclusion of perfect induction just states that men eat, without asserting that humanity is a cause of eating, then it is not a demonstrative proposition, and a fortiori, induction is proof no longer. And if perfect induction is unable to give demonstrative statements, then there is no way to establish ultimate premises of proof.

(3) Perfect induction gives us a judgment about, at most, actually observed instances but not

instances which may exist in the future. We may observe, theoretically speaking, all the instances of man in the past and present and see that they eat, but cannot now observe men that may come in the future. Thus perfect induction cannot give us a strictly universal conclusion. And it makes no difference to make induction dealing with particulars, e.g. John, Peter .. and to arrive at a general conclusion such as every man eats, or dealing with species such as man, horse, lion to judge that all animals die. For a species or genus does not include individuals or species actually existed and observed only, but a species may have other individuals, and genus other species.

(4) Perfect induction has recently been criticized not only as a proof in the Aristotelian sense, but also as a proof in any sense. Suppose I arrived at the conclusion, all matter is subject to gravitation, after a long series of experiments in a great number of instances. Induction maybe formulated thus:

^{a}1' ^{a}2' ^{a}3'…^{a}n are subject to gravitation.

^{a}1' ^{a}2' ^{a}3'…^{a}n are all the kinds of matter that exist.

. . all matter is gravitational.

When I see a piece of stone, I judge that it is subject to this law, not because I give a new judgement, for stones are among the kinds under experiment, but because when I come across some instance not included in my experiments, I judge that the conclusion applies to the new instance as well.

This objection may be retorted on Aristotelian lines. In perfect induction, we do not intend to say that this piece or that piece of stone is subject to gravitation, but that all pieces of matter are so.

Aristotle distinguished syllogism from induction, the former predicates the major term to the minor term by means of the middle term, whereas the latter predicates the major term to the middle term by means of the minor term. Thus, the conclusion that this or that piece of stone has gravitational property is reached not by induction but by a syllogism, formulated thus: these instances have gravitational property; these instances are all matter that exists . . all matter has gravitational property

Further, it should be remarked that the statement all pieces of iron extend by heat is not merely enumerating particular statements expressing the fact this and that piece extend by heat, but it is a different statement from all those particular ones. For the statement all pieces of iron extend by heat is reached by induction in two steps. First, we collect all pieces of iron in the world, separating them from all other species of matter and conclude that these are all iron that exists. Secondly, we turn to every piece of iron and show that each extends by heat. [Only then perfect induction could be properly asserted, reader's note]

Recapitulation

The results reached so far are as follows, (a) The subject of perfect induction does not concern those who consider induction in the modern sense; (b)

Perfect induction can not be regarded as a proof in the Aristotelian sense for it is unable to discover the cause; (c) Perfect induction is formally a valid inference and (d) General statements in science cannot be reached through this sort of induction.

Aristotle's imperfect induction

The Problem of induction

If you ask an ordinary man to explain how we proceed from particular statements to a general inductive conclusion, his answer may be that we face two phenomena in all experiments such as between heat and extension of iron, and since the extension of iron has a natural cause, we naturally conclude from constant relation between heat and extension that heat is the cause, and if so, we have right to make the generalization that when iron is subjected to heat it extends. But this explanation does not satisfy the logician for many reasons. (A) Induction should first establish the causal law [which is an *a priori* principle in rationalistic epistemology, but not in the empiricistic epistemology, which considers empirical observation to be the only source of knowledge, reader's note] among natural phenomena, otherwise extension of iron has probably no cause and may happen spontaneously, and hence another piece of iron may not extend by heat in the future. (B) If induction has got to establish causality in nature, it suggests that the extension of iron has a cause, but has no right to assert off band that the cause is heat just because heat is connected with extension.

Extension of iron must have a cause but it may be something other than heat, heat might have been concomitant with the extension of iron without being its cause [since observation of two adjacent phenomena doesn't necessarily mean that one is the cause of the other, for example in the case of morning following night, nobody says night is the cause of morning, reader's note]. Induction should therefore establish that heat any other is the cause[?]. (C) If induction could establish the principle of causality among natural phenomena, and could also argue that a is the cause of b, it still has to prove that such causal relation will continue to exist in the future, and in all the yet unobserved instances, otherwise the general inductive statement is baseless [the most it could generalize is that heat causes extension in the piece(s) of iron under observation and for that piece(s) of iron only, reader's note].

Aristotelian logic has an answer on logical ground to the second question only; as to the first and the third, it is satisfied with the answers given in the Aristotelian rationalistic epistemology. Rationalism involves the causal principle (every event has a cause), independently of sensible experience. Rationalism involves also the principle that "like causes have like effects" this being a principle deduced from causal principle, and would be the ground of the third question mentioned above. It is the second question only that the Aristotelian logic has got to face and solve, that is, how can we infer the causal relation between any two phenomena that have mere concomitance and not reduce such concomitance to mere chance? To overcome this, Aristotelian logic offers a third rationalistic principle that we now turn to state in detail.

Formal logic and the problem

When a generalisation is through induction, we either apply it to instances which are different in some properties from those we have observed, or apply it to instances that are exactly like those we have observed; the former generalisation, for Formal logic, is logically invalid, because we have no right to infer a general conclusion from premises some of which state some properties unlike the properties stated in other premises. Suppose we observed all animals and found that they move the lower part of their mouth in eating, we cannot generalise this phenomenon to sea animals, since these have different properties from the animals already observed. [2]

But inductive generalisation is logically valid, when applied to like unobserved instances which are similar to instances observed. Validity here is not based on mere enumeration of instances, for this does not prove that there is causal relation between any two phenomena. Formal logic has found a way to assert causal relation in inductive generalisations, if we add, to the observation of instances, a rational *a priori* principle, that is, chance cannot be permanent or repetitious, or between any two phenomena not causally related, concomitance cannot happen all the time or most of the time. Such principle may take a syllogistic form : a and b have been observed together many times, when two phenomena are observed to be severally connected, one is a cause of the other; therefore a is cause of b. This syllogism proceeds from general to particular, and not vice versa, thus not induction.

We then observe that the role played by imperfect induction, for formal logic, is producing a minor premise of a syllogism. This inductive inference involving a sort of syllogism is called by formal logicians an experience, and this is considered a source of knowledge. The difference between experience and imperfect induction is that the latter is merely an enumeration of observed instances, while the former consists of such induction plus the *a priori* principle already stated.

Consequently, it may be said that formal logic regards imperfect induction as a ground of science, if experience as previously defined, is added; that is if we add, to observation of several instances , the *a priori* principle that chance cannot happen permanently and systematically.

Misunderstanding of formal logic

Some modern thinkers mistakenly thought that formal logic rejects inductive generalisations and is interested only in perfect induction. But formal logic showed, as we have seen, that imperfect induction can give logically valid generalisation if we collected, several instances and added a rational principle, such that we reach a syllogism proving causality, and that is called experience [which is also a] [xand a] source of knowledge.

Further, some commentators of formal logic have understood the distinction between imperfect induction and experience in a certain way. Perfect induction is based on passive observation while experience needs active observation. An example of the former is that when we observe a great number of all swans are black. An example of the latter is that when we heat iron and observe that iron extends and conclude that iron extends by heat. This attempt to distinguish induction from experience anticipates the modern conception of experience and makes imperfect induction similar to systematic observation. But this explanation is mistaken, for experience is meant by formal logicians no more than imperfect induction plus the construction of a syllogism, the minor premise of which is based on induction, while the major premise states a rational principle rejecting the repetition of chance happenings.

Aristotelian epistemology and induction

The formal logical view of introducing *a priori* principles in induction is related to rationalistic epistemology which includes that reason independently of sense experience is a source of knowledge. And this theory of knowledge is opposed to the empiricist theory which insists on sense experience as the only source of human knowledge. If we maintain that chance cannot be permanent or repetitious this must be established by induction, and thus that principle is nothing but an empirical generalisation, thus it cannot be regarded as the logical foundation of valid generalisation.

Although we are enthusiast about rationalistic epistemology, as will be shown later, we think that Aristotle's principle (chance cannot be permanent and repetitious) is not an *a priori* principle, but a result of inductive process.

Formal logic and chance

Let us make clear how chance is defined by formal logicians. We may first clarify, "chance", by making clear its opposite, i.e., necessity. Necessity is either logical or empirical. Logical necessity is a relation between two statements or two collections of statements, such that if you deny one of them, then they become contradictory, e.g., logical necessity between Euclidean postulates and theorems. On the other hand, empirical necessity is a causal relation between two things such as between fire and heat, heat and boiling, poison and death; and causality has nothing to do with logical necessity, in the sense that it is not contradictory to assert that fire does not produce heat, and so on. There is a great difference between the statement' the triangle has not three side's and the statement "heat is not a cause of boiling water", The former is self contradictory while the latter is not; necessity between heat and boiling is a matter of fact not a matter of logic.

Let us now turn to chance. To say that something happens by chance is to say that it is neither logically not empirically necessary to happen. Chance is either absolute or relative. Absolute chance is the happening of something without any cause, as the boiling of water without a cause; whereas relative chance is the occurrence of an event as having a cause, but it happens that it is connected with the occurrence with another event by chance, for example, when a Kettle full of water under heat boils, but a glass of water under the zero point freezes; thus it happened by chance that the Kettle boiled at the same time when the glass freezes. Chance here is relative because both boiling and freezing have causes (not by chance) but their concomitance is by chance. Thus, absolute chance is the occurrence of an event without any necessity, logical or empirical - without any cause; where as relative chance is the concomitance of two events without any causal relation between them.

Now, absolute chance for Aristotle, is impossible, for this sort of chance is opposed to the causal principle. Thus, in rejecting absolute chance, Aristotelian epistemology and other sort of rationalism establish the causal principle, and consider it the basis of the answer to the first of our three questions related to the problem of induction; and goes with this the answer to the third question which is deduced from the causal principle. But, for Aristotelian rationalism, relative chance is not impossible, because it is not opposed to causality.

The concomitance between frozen water and boiled water by chance does not exclude that freezing or boiling has a cause. We have in that instance three sorts of concomitance: frozen water and boiled water, freezing, and heat to the zero point, boiling and heat in high temperature; the first being by chance, the latter two are causally related. There is a great difference between concomitance by virtue of causal relation and concomitance by relative chance; the former is uniform and repetitious, such as between the concomitance between heat and boiling, or lightning and thunder. The latter is neither uniform nor recurrent, for example, you for many times, when you go out, you meet a friend, but this does not happen uniformly.

Formal logic takes the previous view as a ground of the principle that chance does not happen permanently or uniformly, considers it *a priori* principle, and by chance is meant relative chance.

Need of definite formulation

Despite clear exposition previously stated, the principle that chance does not happen permanently and uniformly has to be clarified. We ought to know precisely whether the rejection of relative chance applies to all time past, present and future, or is confined to the field of experiments made by some person in a definite stretch of time.

In the former, it follows that relative chance does not recur in all time, but that is impossible since we cannot observe all natural phenomena in the past and future. And if meant by the principle that we reject uniform repetition in the field of experiments made by some person, it follows that the principle seeks to show that relative chance does not recur in a reasonable number of observations and experiments. But the Aristotelian principle has to specify the reasonable number of experiments required. Can we formulate the principle thus: relative chance does not recur in ten or hundred or thousand experiments? Suppose we specified the reasonable number by ten, then if we put some water in a low temperature and it freezes, we cannot discover the causal relation from doing the experiment only once; we have to repeat the experiment ten times, in this case we have right to discover the causal relation.

The crucial point of difference

We differ from formal logic on the principle that chance cannot happen uniformly mainly not its truth but its character. We accept the principle but refuse its being *a priori* and rational nature. Formal logic regards that principle as independent of all sensible experience and then is considered a ground of all inductive inferences; for if it is considered an empirical principle and derived from experience, it cannot be a principle of induction but itself an inductive generalisation. Such principle is, in our view, a result of induction, arrived at through a long chain of observations. Now, the question arises, what evidence formal logic has to maintain that such principle is *a priori*?

In fact, there is no evidence, and formal logic considers the principle as among primitive and primary principles and these do not need evidence or proof. Formal logic divides our knowledge into two sorts; primary and secondary; former is intuitively perceived by the mind such as the law of non-contradiction; but secondary knowledge is deduced from the primitive one, such as the internal angles of a triangle are equal to two right ones. Primitive knowledge needs no proof but secondary sort of knowledge does. But since formal logic regards experience as one of the sources of knowledge, than[?] empirical propositions are primitive.

Since formal logic regards empirical statements as primitive statements, and claims that the principle about chance is primitive, then such principle needs no demonstration, exactly as the principle of non -contradiction need not. Since we have known the definite concept of the principle which rejects relative chance for formal logic, it is now easy to reject that principle. If this Aristotelian principle asserts the impossibility of recurrence of relative chance, as the law of non -contradiction asserts the impossibility of contradiction, we can easily claim that the former principle is not found in us, because we all distinguish the law of non -contradiction from the principle of non -recurrence of relative chance. For, whereas we cannot conceive a contradiction in our world, we can conceive the uniformity of relative chance, though it does not really exist [spurious correlations in social sciences, for example, between number of fire trucks sent to rescue and the destruction caused by the fire. The more the fire trucks, it appears more the fire damage as observed in the recurring events. So is the larger number of fire truck responsible for larger destruction? There is a third variable that actually explains the cause and that is the hugeness of fire. The massive the fire, the more trucks needed every time, and the massive the fire, the more chances of destruction every time]. And if the Aristotelian principle rejects the recurrence of relative chance in our world together with admitting that it is possible to recur, then the principle is not a rational *a priori* principle independent of experience, because *a priori* principles are either necessary or impossible, if it is only possible, how can we reject it independently of sense experience? We have said enough to conclude that the principle of rejecting relative chance is not among *a priori* principles. In the following chapter we shall give a detailed refutation of the *a priori* character of the principle.

Notes

[1]The vision of induction into perfect and imperfect does not seem to be Aristotelian, but was made by later logicians who knew perfect induction and another sort of induction which is now called intuitive induction, but imperfect induction is absent in his writing (Tr.)

[2]This is clearly stated in Avicenna's Isharat and Al-Ghazali's Criterion of Science. (These references are originally Arabic)

Chapter II

Criticism of Aristotelian Induction

In this chapter we continue our discussion of imperfect induction in formal logic, and more particularly a discussion of the principle that relative chance cannot happen permanently and uniformly, being the rational ground of the validity of imperfect induction.

Indefinite Knowledge

The Aristotelian principle rejects the uniform repetition of relative chance in a reasonable number of observations and experiments. Now suppose that such reasonable number is ten; then, the Aristotelian principle means that if there is no causal relation between a and b, and found a ten times, b would be absent once, at least among those ten times, for if b is related to a and those ten times it would mean that relative chance happens in ten times, and that is which the principle rejects. And when the principle shows that any two phenomena not causally related do not come together one time among the ten times, that principle does not specify the experiment in which the two phenomena do not relate; thus the principle involves a sort of knowledge of an indefinite rejection. There are in our ordinary state of affairs instances of knowledge of indefinite rejection: we may know that this sheet of paper is not black (and that is knowledge of definite rejection), but we may know only that the sheet cannot be black and white at the same time (and this is knowledge of indefinite rejection). The sort of knowledge which rejects something in an indefinite (or exact) way may be called indefinite knowledge, and the sort of knowledge which involves a definite rejection of something may be called definite knowledge in consequence, the Aristotelian rejection of relative chance is an instance of indefinite knowledge.

Genesis of indefinite Knowledge

We may easily explain how definite knowledge arises. If you say 'this sheet of paper is not black', this may depend on your seeing it. But if you say of a sheet of paper that you do not know its definite colour, and that it must not be black and white at the same time this means that one of the two colours is absent, and this is due to your not seeing the paper. For if you saw it clearly, you would have specified its colour, then you assert your indefinite knowledge as a result of the law that black and white cannot be attributed to one thing at the same time. Such indefinite knowledge arises in two ways.

First, I begin with the impossibility of conceiving two things to be connected with each other, thus we have indefinite rejection, e.g., I exclude the blackness or whiteness to be predicated of a sheet of paper; this is a result of recognizing that black and white cannot come together in one thing [it can mix together to become grey colour for example, but then it won't be fully black or white which the author meant in the example, reader's note]. Secondly, one may not conceive the impossibility of two things to happen together, but only know that one of them, at least does not exist. Suppose you know that one of the books in your study is absent, but you did not specify the book; here you have knowledge of indefinite rejection; nevertheless there is not such impossibility among the books being put together as that impossibility of black and white being together. Thus our knowledge of indefinite rejection may depend on definite rejection (the loss of a book) without specifying it.

We may now conclude that knowledge of indefinite rejection arises either from conceiving the impossibility of two things coming together, or from definite rejection without specifying it.

Aristotelian principle and indefinite knowledge

The Aristotelian principle of rejecting relative chance, is now shown[?] to be due to a sort of knowledge of indefinite rejection. We have also previously shown that knowledge of indefinite rejection arises from impossibility or from unspecified possibility. Now, we may claim that the rejection of concomitance, at least, in one experiment is an indefinite knowledge on the basis of impossibility, that is, relative chance does not happen in one of those ten experiments. We may also claim that the rejection of concomitance in one experiment at least is an indefinite knowledge on the basis of unspecified possibility, that is, it is definite rejection in fact but unspecified to us. In what follows, we shall try to make clear our position in relation to that Aristotelian principle and deny that it is a rational *a priori* principle and thus not a logical ground of inductive inference.

First Objection

When there is no causal relation between a and b and bring out a in ten consequent experiments, the Aristotelian principle would assert that b is not concomitant with a at least once in those experiments if we take nine the maximum number for recurring relative chances. We maintain that indefinite knowledge of denying at least one relative chance is not explained on the ground of our conceiving impossibility between relative chances, that is, similar concomitance which do not occur owing to causal relation.

For example, suppose we want to examine the effect of a certain drink and whether it causes a headache; we give the drink to a number of people and observe that they all have headache. Here we observe two things, the association of that drink with headache (this is something objective); and a random choice by the experiments (this is something subjective). If there is really a causal relation between the drink and headache, these two associations are natural result of that relation, and there is no relative chance. But if we know already that there is no causal relation, then there is relative chance; we then [???] whether relative chance apply to objective concomitance between drink and headache or subjective concomitance between random choice of instances and headache.

It is possible that I consciously choose those persons susceptible for headache and subject them to experiment, and then I get a positive result which actually happened by relative chance. It is also possible that random choice is associated with headache. For suppose that relative chance would not be repeated ten times, the experimenter may choose randomly nine persons, but if so, he would be unable to choose randomly any of those persons since relative chance cannot occur ten times.

It is not the number of relative chances that is important, but their comprehension of all the instances which belongs to one of the two phenomena. When we have two phenomena a and b and observe[d] that all the instances belonging to a are concomitant with b, it is impossible that the concomitance between b and a is by chance. But if we observed that a limited number of instances belonging to a is concomitant with b, it is not impossible to have connected by chance.

We may face three phenomena a, b and c; when all instances of c are concomitant with b which are at the same time members of a, but we know nothing of the concomitance of other instances of a with b, then if you suppose that c is not a cause of b, we may conclude that a is cause of b, and say: all a is connected with b. Now, we may get an explanation of inductive inference under two conditions:

(a) Complete concomitance in the sense that we add c to a and b, and that the observed instances of b would be all instances of c, but not all instances of a.

(b) Previous knowledge that c is not causally related to b. When these conditions are fulfilled, we have two alternatives either a is cause of b and then no chance of b, and then c and b are concomitant by chance. But our discussion excludes complete chance, thus, a is cause of b.

Second objection

In every instance which involves incompatible things, we may utter hypothetical statement, namely, even if all factors for those things are coexist, they never do so by reason of their incompatibility. Suppose a room is too small to gather ten persons, then even if all of them are to enter that room, they could not. Now, concerning the possible repetition of relative chance, we are certain that such chance cannot recur uniformly. If you randomly choose a number of persons and give them a drink, we are sure that they would have headache by chance, but at the same time we cannot apply the previous hypothetical statement.

Now, though we believe that relative chances do not occur regularly and uniformly, we cannot assert that they should not occur. Thus our assurance that the concomitance between having a certain drink and headache cannot be repeated uniformly does not arise from the incompatibility of such concomitances.

Third Objection

We try to show in this objection that the indefinite knowledge on which the Aristotelian principle is based does not depend on probability. So, we must recognise that any indefinite knowledge is a result of the occurrence of a positive or a negative fact, but that indefinite knowledge of such fact depends on our confusing a fact with another. For example, if we are told by a trustworthy person that someone is dead and called his name but I could not hear the name clearly; in such a case we have an indefinite knowledge that at least one person died, that such knowledge is related to the fact of a certain death but the fact is said vaguely. Thus indefinite knowledge, resting on hesitation or unclear information, is related to a definite fact referred to vaguely, and any doubt about it causes such knowledge vanish.

Now, taking notice of what formal logic says of relative chance and that it cannot recur consistently through time, we find that indefinite knowledge of this is not related to denying any chance in fact, and this means that the indefinite knowledge, that at least one instance of relative chances did not occur, does not rest on hesitation or probability. Chance happening which can be referred to vaguely is not a ground of indefinite knowledge, while the event of death which is referred to vaguely is a ground of the indefinite knowledge that someone is dead. Thus, we think that indefinite knowledge of the non-occurrence of at least one chance does not vanish even if we doubt in any chance referred to vaguely.

Fourth Objection

Here we try to reject the idea of *a priori* indefinite knowledge based on analogy and hesitation. That is, we try to argue that the knowledge of the non-occurrence of chance at least one out of ten times is not an *a priori* indefinite knowledge. To begin with, we wish to define *a priori* science for formal logic. There are two sorts of *a priori* science in formal logic; ultimate rational sciences including ultimate beginnings of human knowledge, and rational sciences derived from those, and deduced from them. [a priori science or a priori knowledge; and, is it primary rational knowledge vs. secondary knowledge??? Translation problems]

Both have a common basis, namely, that the predicate is attached to subject of necessity. It is not sufficient, in order for a science to be *a priori*, to attribute something to a subject but they must be attributed necessarily.

This necessity is either derived from the nature of the subject or issued from a cause of the relation between subject and predicate. In the former, the statement is ultimate, and our knowledge of it is *a priori* of the first sort. If the terms are causally related, the statement is deduced, and our knowledge of it is *a priori* of the second sort. And the cause is called by formal logic the middle term. For example, the indefinite knowledge that a headache cannot occur By chance at least once in ten cases cannot be *a priori* knowledge, as formal logic is ready to claim. Such indefinite knowledge, if it rests on analogy and hesitation, is related to a chance in fact. We know that something really happened but we are unable to specify it.

Now, we may argue that such knowledge is not *a priori* since we do not know whether this chance did not happen or it is necessary not to occur. If such knowledge means just the non -occurrence of the chance happening, then it is not *a priori* knowledge, since this involves a necessity between its terms. Whereas if such knowledge means the necessity of its non-occurrence, then such necessity is out of place in a table of chance. If we know that someone who had a drink, had a headache ten regular times, then we have no reason to deny that he got headache in any one of these ten times. But we supposed his feeling of headache for no sufficient reason, we believe that headache had not occurred to him in one of those ten times. Thus the knowledge of the non-occurrence of headache in some cases does not arise of *a priori* idea of the cause, just because we do not know the causes of headache.

Fifth Objection

Formal logic is mistaken in claiming that indefinite knowledge of regular recurrence of chance is *a priori* knowledge. For it says it of indefinite knowledge that if there is no causal relation between (a) and (b), then there is uniform concomitance between them. Suppose such concomitance to be ten successive occurrences, we may conclude that (a) is cause of (b) if ten times succession is fulfilled. For example, if (a) is a substance supposed to increase headache, (b) the increase of headache, and ten headached-persons got the treatment and they got more pain, we conclude that regular relation between (a) and (b) is causal and not by chance. Suppose we later discovered that one of the ten persons had got a tablet of aspirin, without our knowing it; this discovery will falsify our test and our experiment was made really on nine persons only. And if ten experiments are the minimum of reaching an inductive conclusion, then we have got no knowledge of causal relation in that experiment.

Thus, any experiment will be insignificant if we realise that besides (a) and (b) (supposed to be causally related) there is some other factor which we had not taken notice of during the experiment. Thus, formal logic fails to explain these facts within its theory of justifying induction, which presupposes indefinite knowledge that chance cannot recur uniformly. For if such *a priori* indefinite knowledge were the basis of inductive inference and discovering a causal relation between (a) and (b), our knowledge of causality would not have been doubted by our discovering a third factor with (a) and (b). This discovery denotes the occurrence of one chance only, and this does not refute our *a priori* knowledge, supposed by formal logic, that chance cannot recur regularly in the long run.

The only correct explanation of such situation is that knowledge that chance does not happen at least once is a result of grouping a number of probabilities: the probability of the non-occurrence of chance in the first example, in the second,... etc. If one of these probabilities is not realised, i.e., if we discover a chance happening even once, we no longer have knowledge of such probabilities. And this means that this knowledge is not *a priori*.

Sixth Objection

When we start an experiment to produce (a) and (b), and think of the sort of relation between them; we are either sure that (c) does not occur as cause, or we think that its occurrence or non-occurrence is indifferent to the production of (b). Concerning the first probability, formal logic is convinced of (a) being the cause of (b), since (c) does not occur. Then we need not, for formal logic, repeat the experiment. On the other hand, we may find that our knowledge of causality in this case depends on repeating the experiment and find the causal relation between (a) and (b). The reason for this is to make sure of the effect of (c); that is, the more (c) occurs, the less (a) is believed to be the cause, and vice versa.

This means that inductive inference of the causal relation between (a) and (b) is inversely proportional to the number of cases in which (c) occurs. Thus, unless we have *a priori* knowledge that (b) has a different cause in nature, we tend to confirm the causal relation of [???] and (b). For the probability of the occurrence of (c) is low. The connection between inductive inference to causal relation and the number of the probabilities of (c) occurring in many experiments cannot be explained by formal logic. For if induction is claimed to be a result of an *a priori* ultimate knowledge that there is no relative chance, then the more we get concomitance between two events, we conclude the causal relation between them, minimizing the effect of the occurrence or non-occurrence of (c).

Seventh Objection

If we assume that the long run, in which we claim that relative chance does not recur, is represented by ten successful experiments, then the concomitance between drink and feeling of headache in nine successive experiments is probable, but not probable if the concomitance happens in ten successive experiments.

Now, we try to argue that such knowledge is not an immediate datum given *a priori*. First every *a priori* rational knowledge of something necessarily implies *a priori* knowledge of its consequence. Secondly, if it is true that relative chance cannot uniformly recur rational statement. The problem of the probability of absolute chance is overcome by assuming the principle of causality. The problem of the probability of relative chance is overcome by denying its uniform recurrence in the long run. The problem of doubling uniformity in nature is finally overcome by assuming a statement derived from causality, namely, like cases give like results.

Such situation may be summarised in two points. First, formal logic maintains that inductive inference requires three postulates to meet its three problems, thus acquires the desired generalisation. If these postulates are shaken, inductive science collapses. Second, formal logic maintains that the principle of causality, the denial of the recurrence of relative chance, and the statement that like cases give like results are all *a priori* rational statements independent of experience. Hence, its postulates are accepted.

Our previous discussion was confined so far to only one of those three statements, namely, the denial of relative chance. We have concluded that such statement is not *a priori*; it cannot work as a postulate of induction. In our view, formal logic is mistaken not only in regarding such statement *a priori*, but also in claiming that inductive inference needs *a priori* postulates. We shall later see in this book that induction may work without any *a priori* postulates, that postulates, given by formal logic may themselves be acquired by induction.

Chapter 3. Induction And Empiricism

The rationalistic theory of induction and its justification had been sketched in the previous chapters. Now, we turn to discuss the empiricist theory of induction, and its solution to the problems involved from a logical point of view. By empiricism is meant the assertion that experience is the source of all human knowledge and the refusal of any *a priori* knowledge independently of sensible experience. Inductive inference faces, as has already been noted, three main problems; (1) why should we suppose a cause of (b), excluding absolute chance for its occurrence? (2) if there is a cause of (b), why should we suppose that (a) is its cause being concomitant with it, and not supposing that (b) is connected with (c) by relative chance? (3) If we could make sure, by inductive process, that (a) is the cause of (b), on what ground can we generalise the conclusion that all a's would be causes of b's? Formal logic solved the first and third questions by appealing to certain *a priori* principles on rationalistic lines, and solved the second problem by supposing another *a priori* principle denying the systematic repetition of relative chance.

Since empiricism refuses any *a priori* principles and any rationalistic interpretation of causality, it was mainly concerned with giving different answers to the first and third questions, giving the second question a minor importance. We may make the empiricist theory clear if we distinguish three attitudes: certainty attitude, probability attitude, and psychological attitude.

Certainty Attitude

J.S. Mill, one of the greatest British empiricists, may be a representative of this attitude because he held the view that inductive conclusions are certainly true. His views on induction may be clarified if we summarise his positions on the first and third questions, then on the second question, already referred to.

On the First and Third Questions

Certainty attitude holds that inductive inference has two notions as its ground, namely, causality and uniformity in nature, these are first premises. Like causes have like effects. It agrees also with formal logic in regarding inductive inference as a syllogism, the minor premise of which expresses particular instances, and its major premise expresses those two beliefs in causality and uniformity. It will be found for example that the extension of iron is, in all cases, concomitant with the occurrence of heat, and it is then concluded that when a phenomenon occurs under certain conditions, it occurs all the time in these conditions.

Further, empiricism differs from rationalism in considering causality. For whereas the latter takes causality as *a priori* principle, the former reduces it to sensible experience, that is why Mill maintains that our belief in causality is the outcome of widespread inductions in the physical world. We derive our concept of cause from induction, but once so derived, this concept becomes our basis of any subsequent inductive generalization.

Empiricism differs also from rationalism in detecting the meaning of cause; the rationalistic meaning of cause involves a <u>necessary</u> relation between certain phenomena, such as that when a phenomenon produces another, then the first is the cause, the second the effect. Whereas empiricism means by causal principle that every event has a cause in nature, but there is here <u>no</u> necessity [or production] since these go beyond experience. Causality means, for Mill, nothing more than a certain succession between two events. But not every temporal succession is causal; causal succession requires that succession should be uniform.

We may now compare rationalistic conception of cause to empiricist one. First, succession between cause and effect is temporal in the latter, whereas it is ontological in the former. Secondly, rationalism takes causal relation as uniform concomitance between two events, resulting from the existence of a cause and its production by necessity of the effect, this being deduced from its cause. But empiricism refuses causal relation to be uniform concomitance and holds it to be a relation of another kind, namely, the observation and not deduction of temporal succession.

Discussion

Our comments on certainty attitude concerning induction are as follows. First, the author differs from both rationalistic and empiricist answers to the second question, namely, whether induction needs causality as a necessary postulate. Both schools, though different, answer that question in the positive, while the author will say no, owing to what will be maintained in the sequel. Secondly, we agree with certainty attitude that causal principle is itself reached by induction, and thus hold that induction needs no *a priori* postulates. But the impasse of empiricism in our view, is that it holds that induction is grounded upon causality postulate while it holds at the same time that causal principle is itself an inductive generalisation.

If we reach causality by induction, why need we any *a priori* postulate to vindicate induction? Finally, when we say that induction is the basis of our idea of causality without any *a priori* postulate, we understand causality in the rationalistic sense as expressing a relation between cause and effect; otherwise, we cannot argue that causality is a result of empirical generalization. For the principal condition on which induction depends is, in our view, to conceive causality in the rationalistic sense, and if this is not the case, inductive inference would be incapable of reaching any generalization whatsoever, even in the probabilistic level.

On the second question

It is now possible to state the empiricist answer to the second problem of induction, which was central to formal logic as we have seen. For empiricism maintains our belief in the uniformity of nature, that is, if two certain events succeed each other under certain circumstances, we believe succession to occur in the future. It is meant of course, not that the belief is maintained if these two events happened once or twice, but that those events should have occurred countless times; and then we reach the same rationalistic position that relative chance cannot recur uniformly, with the difference that the latter position depends on *a priori* statement while empiricism rests on inductive inference in reaching uniformity.

Answer to that question

J.S. Mill has given us in his inductive logic four methods to discover the causal relation between any two phenomena. These methods are concerned with the second problem of induction, rejecting relative chance in nature.

These methods are stated as follows: (1) The method of agreement: If two or more instances of the phenomenon under investigation have only one circumstance in common, the circumstance in which alone all the instances agree is the cause (or effect) of the given phenomenon"[3]. For example, if the phenomenon to the explained (b), is preceded or succeeded in the first instance by the circumstances (a), (c), (d), in the second by (e), (f), (a), and in the third by (x), (a), (y), then the only circumstance in common (a) it is cause of (b).

If we wish to discuss this method deeply, we may discover clearly that it deals in fact with the problem of the probability of relative chance. In the first instance of the phenomenon (b), we find that (b) succeeds (a), but there is still the probability that (b) is caused by (a), or by (c), or (d), where as we find in the other instances that the probability is greater in the connection of (a) and (b) than otherwise.

Thus the role of this first method is confined to facing the second problem of induction, and enables us to minimize relative chance.

(2) The method of difference: "If an instance in which the phenomenon under investigation occurs, and an instance in which it does not occur, have

every circumstance in common save one, that one occurring only in the former; the circumstance in which alone the two instances differ is the effect, or the cause, or an indispensable part of the cause, of the phenomenon"[4]

Such method, like the first, is designed for solving the second problem and decreasing the probability of relative chance. For when we encounter the first instance of the phenomenon (a), and try a variety of circumstances, we cannot ascribe causality to only one of those circumstances; so that we cannot say A is the cause. And when we come to another circumstance, we get the same result. So we tend to make [A and a] causally related, and no relative chance involved.

(3) Method of concomitant variations: "Whatever phenomenon varies in any manner, wherever another phenomenon varies in some particular manner, is either a cause or an effect of that phenomenon, or is connected with it through some fact of causation"[5].. If we have two phenomena, and studied one of them in various circumstances, then we find that it occurs in different degrees, and when in studying the other phenomenon we find that the variations happened to it correspond to those happened to the former phenomenon, then there is causal relation between the two phenomena.

This method, we notice, is nothing but a complex form of the method of agreement, for the third method involves a circumstance in common among various instances. Yet, the third method adds that such common circumstance has different degrees.

(4) Method of residues: "Subduct from any phenomenon such part as is known by previous inductions to be the effect of certain antecedents, and the residue of the phenomenon is the effect of the remaining antecedents".

This method, it is said, enabled astronomers to discover, theoretically, the planet "Neptune". For they held the theory of gravitation to be true, and observed some diversion in the orbit of the planet, "Uranus", contrary to what gravitational theory prescribes. Such difference between theory and fact needed an explanation, so Liverier put forward the hypothesis that such diversion in the orbit of Uranus is due to interference of some yet unknown planet. Later, it was discovered and called Neptune.

Although this method is badly formulated, we may put it aright with application to the discovery of Neptune; this may be put as follows. When astronomers observed the diversion of Uranus from its normal orbit according to gravitational theory, they provided two alternatives to explain this: either to suppose the existence of a new planet which causes such diversion if the theory of gravitation is true, or that no new planet and then the theory is defective. Astronomers preferred the former, on the ground that very many other phenomena have confirmed the theory of gravitation, and then we tend to rule put the supposition that the diversion of Uranus' orbit happened by chance.

We may conclude that Mill's four methods are intended to consider the second of the three problems of induction, in opposition to relative chance. Formal logic put forward the principle "chance cannot occur permanently and consistently", while Mill provided his four canons to oppose complete chance. Nevertheless, Mill had not succeeded in rejecting chance occurrence of phenomena in all respects, [x] since he made probable other circumstances to produce the effect other than the assigned cause.

Probability Attitude

This is the second attitude of the empiricistic theory of induction. Such attitude suggests that inductive generalisation needs certain assumption and postulates which can be confirmed independently of induction itself; but it maintains also that such confirmation is not possible on rationalistic lines since it rejects any *a priori* principles, not is it possible in accordance with certainty attitude which believed such postulates to be considered as results of previous inductions. And since those postulates cannot possibly be confirmed, inductive conclusions cannot be certain, but only probable: any more observation or experiment helps to increase probability of the conclusion.

It may be useful to quote from Professor Zaki Naguib, a proponent of probability attitude, the following: "The majority of those interested in induction, including a rational principle not derived from sense experience, as our ground of generalising (scientific judgments). Even if you are enthusiast empiricist, you have to confess that there is something not derived from experience, namely, that what applies to some instances of a kind equally applies to all the instances belonging to it; hence our generalizations. Thus, Russell holds that we are obliged at the end to rest in induction to an unempirical basis, the so-called principle of induction. (Those who consider induction as the only scientific method think that all logic is empirical, and it is not expected of them to hold that induction requires a logical principle improvable by induction itself; such principle must be *a priori*).

Now, most logicians, including Russell, maintain that experience alone is not sufficient and then we either accept the principle of induction as indebatable assumption, or seek in vain for a justification of predicating future phenomena from the present".

"The question naturally arises, how can we judge the validity of inferring future events from past ones without recourse to any rational principle" such as the principle of induction suggested by Russell? Or is there a justification that new experiment be similar to past ones? In defence of empiricism, we [main turn???] ask: what is meant by rational justification? This may mean that the conclusion is certainly true, or that induction is to be considered as deduction the conclusion of which be implied in its premises ... Induction in this sense has no rational justification, for induction is no deduction".

"But such meaning is accepted neither in the sciences nor in ordinary way of speaking. If I was told one day that (a) will play chess with (b), and all that I know about them is that both had played six times in the past, and (a) won the game in four times out of six, while (b) won twice, then I am justified in saying that the probability of a's winning is expected. Similarly, it is more probable that a falling body may come to the ground, the sun will rise tomorrow [than not] [???]. That is probability not certainty; but that is what the sciences are prepared to accept, because certainty is not expected save in mathematical propositions'[6]

Discussions

(1) We maintain that induction paves the way to scientific generalisations, but we wish to regard it not as deduction, but another sort of inference which proceeds from particular to general without need of any *a priori* principles. This topic will be considered at length in a later chapter, let it suffice now to say that we shall never prove that a normal person know by induction a great number of generalisations; we can have no proof to convince such person of this knowledge. How can we convince someone that if he eats he becomes hungry no longer, when he denies such process? Such person is similar to the idealist who denies the existence of the external world or any objective reality outside his own concepts. We cannot convince the idealist that he approves of the objective reality of his family, even if we are sure of

that since he lives with them and other people. It is the same with the person who denies his knowledge of ordinary generalisations.

We may distinguish three sorts of certainty: logical, subjective and objective; the first concerns deductive inference; the second is a personal affair, while objective certainty concerns induction, and the last certainty is not proved but only postulated and explained. Further, it is absurd to find a rational justification of induction if by this is meant deductive process involving the law [of] non-contradiction. For inductive conclusion cannot be logically certain.

On the other hand, if it is meant by justification the claim that the negation of inductive conclusion is probable, then this is an important claim since it involves that inductive conclusion cannot be a postulate. We shall later be concerned with the condition of reasonable postulates.

(2) If we follow the empiricistic line of thought we shall not reject only inductive science but also any degree of probability for inductive inference. The representative quotation mentioned above shows that such probability depends on probability calculus, but it will be shown later that such calculus does not lead to increasing probability of inductive conclusion unless it does also lead to confirming the rationalistic conception of causality, but empiricism rejects such conception. Thus, empirical logic faces a dilemma, either it leaves the empiricistic conception of causality and adopts the

rationalistic conception, or it excludes the rationalistic conception of causality and insists on the empiricistic conception, but then it will be unable to explain the probability of induction.

Psychological Attitude

We mean by this attitude, an empiricistic one that deprives induction of any objective validity but connects it with habit; and David Hume is a clear representative. Modern Behaviourism, a great modern school of psychology, later come to continue the Human tradition and transfer it from the field of philosophy to that of psychology.

Hume tries to clarify the problem of induction as follows. All inference concerning matters of fact is based on causal relation, and this relation is the sole relation which goes beyond the senses and informs us of entities that we do not perceive. If you ask some one about the cause of his belief in any absent matter of fact, he would justify such belief by means of his knowledge of another fact causally connected with it, he would say that he believes that x is sick because he saw a doctor visiting him. Or that if he was going yesterday to throw himself into fire, we are justified to say that he may be burnt, because there is a causal relation between thrown into fire and being burnt. We may now ask, how do we know such relation? The source of such knowledge is experience which enables us to observe the concomitance between both events. We may further ask, how do we know that actual concomitance between two events will happen in the future? We are in need of a justification of the principle that the future will be similar to the past. That was Hume's formulation of the problem. He solves the problem by saying that the justification of uniformity in nature is not logical but psychological and this can be done by giving analysis of cause-effect relation.

Perceptions are either impressions or ideas for Hume, and these are distinguished by virtue of the degree of vividness connected with impressions. And impressions include all sensations, emotions and sentiments. Whereas ideas are fading copies of impressions when the object of perception is absent. For instance, when we look at the sea, we perceive it vividly and clearly and here we get an impression, but when we turn our back to it, we get an idea, a copy of such impression. Hume then maintains that impressions are prior to ideas, that every idea, simple or complex, has, for its origin, an impression.

Impressions for Hume, are of two kinds, impressions of sense and those of reflection. When we see a lion we obtain an impressions of it, which is clear and vivid, and when the lion disappears, the mind is able to keep an idea of the beast. Such idea produces in the mind fear and aversion, and these may be called impressions. And the operation by which we recall our first impressions is memory, our second impressions is imagination. Ideas of memory differ from ideas of imagination in clarity and vividness as well as their being literal copies of the original impressions, whereas imagination is free. However, freedom of imagination does not involve invention but is somehow derived from a previous impression; it is free in the sense that it is able to manipulate various ideas and construct out of them what it will.

Among those ideas produced by impression, there are certain relations which make the mind proceeds from an idea to another, and such is called association. Relations of association are three; likeness, contiguity in place or time, and causality, the last being the most important. For it involves only one of the two terms of the relation; for instance, when I put water on fire, the causal relation stirs in me the idea of the heat, though I get no impression of such heat; I get an impression of only one of its terms, i.e., water being on fire. The case is different with relation of likeness and contiguity, because they make the mind pass from on idea to another like it or contiguous to it.

Now, we may ask, what gives rise in our mind to the idea of cause and effect? What impression gives rise to our idea of cause? And Hume replies that the idea of cause means not merely the spatial or temporal contiguity of two phenomena, but necessity. But from what impression can we get the idea of necessity. Suppose we saw the event concomitant with another event (b) once, then we cannot say with certainty that there is a relation between them.

But suppose that such concomitance has happened several times, then we are justified in claiming a causal relation. Thus repetition and coexistence is the source of our idea of necessity. Hume clarifies his position by saying that repetition and coexistence is not the source of necessity, but necessity comes from an extra impression, that is, readiness of the mind to pass from one thing to another usually accompanying it.

Such is the nature of necessity involved in the causal relation; it is something in mind not in things; however, we are disposed to apply it to things outside us and think that all events do have such relation among them. Now, Hume could explain inductive inference and its jump from particular to universal in a subjective manner based on, mental habit and psychological necessity, not objectively in term of external reality. This does not mean that he doubted inductive conclusions and in proposition based on experience; on the contrary, he believed in them. But what does he mean by belief? Belief is an idea involving vividness and force. As has already been said, Hume makes impressions more vivid and clearer than ideas. Now, certain ideas may obtain such vividness and clarity and thus become beliefs; the main difference between belief and imagination is that the former is an idea which acquires the vividness and force of impression, while the latter is an idea which does not.

Such vividness and force of belief depend on two things: (a) there being a vivid impression of something (or an idea of memory which has enjoyed the vividness of impressions); (b) the concomitance of this something with some other thing, then the mind passes from one to the other.

Examination of psychological attitude

(1) Belief

Hume explains what is meant by belief in two ways:

(A) The difference between idea and belief is not in content but in our way of recognising them. If there is an idea and we judge that the object of such idea exists, then we have two different things, the difference is not known by recognising elements in the second case which are absent in the first, thus the idea of existence is not different from something existing. Hence existence is not an attribute added to other attributes of an object of perception. And our belief in the existence of something has nothing added to the mere existence of it. Nevertheless, there is clear distinction between identifying an idea of an object in my mind and my belief that such object exists. Since this distinction is not a constituent of the contents of the idea, then it is a product of the way of identifying it.

(B) This distinction is due to the advent of idea in our mind in a forceful and vivid way; if the idea is feeble it is not a belief. Thus belief may be defined as a vivid and forceful idea.

In discussion of Hume's theory of belief, we observe two things. First, we agree with Hume that a belief differs from an idea not in having an element of existence to its content, but in our way of identifying it. But we disagree in justifying such difference. In claiming that existence is not a characteristic of belief, we see that it may be an element of the content of idea as well; if we do not believe that there is a bird with two heads, then we may have an idea of it, and may further conceive its existence without our believing it. Therefore, the element of existence may be included in both idea and object, and we have to find out a characteristic which distinguishes them, it is our way of perceiving an object which gives belief. Hume arrived at the same conclusion from a different premise, namely, that existence is not an element added to the properties of the object of perception. This is based on his principle that an idea is necessarily a copy of an impression, and since there is no impression from which the idea of existence is derived, Hume has to say that existence is not a distinct idea.

Secondly, we observe that certain ideas may be vivid and effective without being beliefs, such as that which we get from illusion; a stick the lower part of which is immersed in water is observed as if it were broken. It may be objected that our belief in the straightness of stick is due to discovery of visual illusion and with the help of actual impressions: thus such belief is nothing but an idea possessed of vitality and force. The objection does not alter our position that the idea of straightness of the stick is vivid without being a belief.

Further, in explaining belief as vivid idea, Hume supposes that this vividness is derived from an impression, either directly as copy, or indirectly as causally related; and this means that any idea not derived form impression is not a belief. But this does not accord with reality, because we may have a number of beliefs without there being copies of impressions. How could Hume explain someone's belief that a ghost frightens him, so long as he had not received an impression of ghost? We must distinguish two things in matters of belief, explanation and evaluation. To explain it we have to distinguish an idea from a belief. And we should give an explanation which applies to all beliefs, regardless of judging it true or false.

(2) Causality and Reason

Hume maintains that causal principle does not arise in pure reason, and cannot be deduced from the law of non-contradiction, it is known to us through experience and not *a priori*. If something as cause has not happened in connection with some other thing as effect in experience, we cannot perceive causal relation. "If we suppose Adamto have perfect mental acts" Hume says, "he could not have deduced from liquidity and transparence of water that he would suffocate when he sinks in it"[7]. We have to distinguish causal principle from causal relations among events. By the former, we mean that every event has a cause, by the latter the relations of heat to extension, boiling to evaporation, eating to nourishment. Rationalism claims that causal principle is known *a priori*, and that causal relation is perceived *a priori*. Aristotelians, among rationalists, maintain that our knowledge of the principle is *a priori* and not derived from experience, while our knowledge that heat is the cause of extension of iron is so derived.

Thus, concerning causal relations, Hume and Aristotelians agree. Let us now discuss Hume position of our knowledge of causal principle. We agree with Hume that this principle is not deduced from the law of non-contradiction, for there is no contradiction in the occurrence of an event without any cause. Now there are two attempts to defend the view of medieval Aristotelians that causal principle is unempirical.

The first attempt may be stated as follows. All events are contingent; by contingency is meant that existence and non-existence are equally possible. Then, for an event to occur, there must be something having the power of giving existence to it the event, rather than not, and this something is the cause. This argument, if discussed thoroughly, is merely a deduction of cause from itself, thus it is a petty principle or that the argument presupposes causality.

The second attempt way formulated thus.

(a) Every essence is possible by itself, and does not exist unless something pushed its existence by necessity.

(b) Every possible essence must not exist except by virtue of an external cause, because its possible existence means that its existence and non-existence are equally acceptable. By necessity is meant that its existence is more probable than non-existence.

Therefore, since necessity of possible essence does not arise except by virtue of an external cause, then it cannot come into existence except by means of a cause. This argument, like the former, is formally invalid because it uses causality as premise, which is meant to be proved.

We suggest that if rationalism is to defend the causal principle as *a priori*, it should claim that the principle is an ultimate proposition in the mind, instead of saying that it is logically deduced from ultimate principles, and thus it becomes impossible to deduce an ultimate proposition from another. Naturally, Hume would reject the suggestion but we shall have occasion later to defend it.

(3) Causality and Experience

When Hume maintained that causal relations cannot be deductively inferred one from another, he claimed also that they cannot be empirically inferred. For all knowledge of the external world is derived from sensible impressions, and we get no impression of causal relation as necessary, we never get an impression of something that can be deduced from impression of anther. All that is acquired from experience is that the effect follows its cause as a matter of fact: if moving billiard-ball is seen to come across another at rest, this is seen to move, so that our sight is affected in such a way that a moving ball is succeeded by another moving ball[8]. Thus, Hume concluded hat causal relation can only be given a psychological, not a logical empirical, explanation: no necessity between food and nutrition, but constant connection between our ideas of both.

Hume's rejection of causality as involving necessity raises two questions, namely, how can we understand the idea of causality as involving necessity since every idea is to be a copy of impression? and how do we believe in causality as objective relation between any two events independently of experience? Hume answered the former by showing that our idea of causality derives from an impression preceding another. And he admitted that the latter involves real problem and claimed that the causal relation is subjective not objective, that is, it lies in our mind as a relation between two ideas not between two events in the world.

If we accept Hume's answer of the first question, then we admit our having the concept of causality. We may then ask whether causality has an objective reality? Although we get, in Hume's opinion, the idea of cause from an impression of connection, there is no reason preventing us of asking whether such idea has objective reality.

Suppose, [???]with Hume, that we cannot get the idea of causality by pure reason, we cannot prove that pure reason refutes causality; that is to say, it is probable that every event must have a cause, since it cannot be confirmed or refuted by reason[9]. On the other hand, we wish to ask whether there is any empirical evidence for the probability of the proposition about causality in objective reality. Hume maintained that there is none, but we shall have occasion later to show such probability.

(4) Concept of Causality

For Hume, every simple idea is a copy of an impression, and when he tries in vain to find out an impression of causality which involves logical necessity, he supposed an impression derived from a connection among certain ideas of succession, and saw that the succession of such ideas stirs in the mind a certain impression of expectation, when we get an impression of the first event we expect the other to occur. It is noticed here that such mental expectation is inferred from Hume's dictum that impressions are prior to ideas, but we see that this dictum is based on induction.

For all simple ideas, in Hume's opinion, are similar to simple impression, and though the idea of causality is not reached by induction, it must have been reached, as idea, by induction. Now, if inductive inference has no objective value, as Hume claims, so is causality. But Hume's application of induction to our idea of cause is invalid, because successful generalisation should not apply to those kinds of things which have specific differences with other kinds. For example, if we find by induction that all metals except gold extend by heat, we cannot include gold in our generalisation for its specific difference from iron, copper etc. Our idea of causality is similar to gold in this respect: if we find that all ideas are preceded by impressions, we cannot say this of our idea of cause which is specifically different from other ideas.

(5) Belief in causality

Let us first state Hume's theory of belief. It is an idea having a high degree of vividness and strength which is derived from a vivid impression or another idea.

When we have two ideas involving causal relation, the former being vivid is belief, and when we move from the idea of cause to that of effect, this requires a similar one once idea is related to an impression it becomes a belief, and if an idea is not so related it requires two things in order to become a belief: (a) a certain relation with another idea which enables us to proceed from the one to the other; such mental habit results from repeated concomitance between any two events in experience; (b) that the other idea should be also vivid. Now, we may criticize such theory by bringing the following points.

(a) Belief in causality involves two propositions, one of which is categorical, e.g., iron extends in heat, the other is hypothetical such as iron extends if exposed to heat. But whereas Hume's theory of belief explains our belief in the former proposition, it does not explain our belief in the latter. For which idea could be taken to be belief in the hypothetical proposition? Is it our idea of the extension of iron or that of heat as cause of extension? Hume cannot have chosen the first answer, because our idea of the extension of iron cannot be a belief unless it acquires a high degree of vividness from its relation to our idea of heat, but our idea of heat has no such degree since it did not occur in fact. The idea of cause, in the case of hypothetical proposition, is not vivid but a mere hypothesis and thus cannot be a belief.

On the other hand, Hume cannot accept the answer that the belief which we possess, in the case of giving a hypothetical proposition, is that heat is the cause of extension. For Hume denies that there are causal relations in objective reality and claims only that such relations are among our ideas. Then when we supply a hypothetical proposition involving causality, we mean, for Hume, the mental habit which helps to proceed from the idea of heat to that of extension.

When we reflect on this, we find that we talk not about the future of events in reality, but the future of our mental habits. Thus Hume cannot on his theory explain our belief in causality in the course of future events. In other words, if we have the right to suppose future to be similar to the past and present, we may apply his supposition to objective reality. But if, as Hume insists, we are not justified in our belief in uniformity, we have no right to talk about the future of mental habits. We may conclude that Hume's theory fails to explain inductive inference which provides hypothetical as well as categorical statements.

(b) Hume gives a ground to establish his theory of inductive inference and causality. He asks, why we need innumerable cases to reach a conclusion without being satisfied with only a small number of cases; and he gives his answer. Whereas the conclusion which the mind reaches from contemplating one circle are the same as when we see a number of circles, we cannot conclude from seeing only one body moving by impulse that all bodies, move by impulse. For in the latter case we require repetition of concomitance between those objects, thus we acquire a habit of inferring the one from the other so that all inferences in experience are effects of habit not by reasoning[10].

But we may explain the validity of inductive inference including our beliefs in causality and uniformity without recourse to Hume's theory. Suppose we observe an event (a) followed once by another event (b), we may say that their concomitance happened by chance, and that (b) is caused by a yet unknown event (c). But if (a) is always followed by (b) in such a way that wherever (a) occurs, (b) follows, then chance is eliminated; repetition and absence of exception is a basis of inductive inference, not in terms of mental habits but in terms of objective reality, as known in probability calculus.

(c) Suppose someone tried to find out the effect of a certain drug on people having a certain disease, and found that this drug gives rise to some physiological phenomenon, then he would conclude that the drug is the cause of that phenomenon. Suppose, further, that such a person discovered that his partner intended to mislead him by providing those cases susceptible for such phenomenon, then our person would give up claiming a causal relation between the drug and that phenomenon. Here we are entitled to admit the objective reality of cause relations in the course of events apart from our ideas, because our experiment involves that certain phenomena should have as yet undiscovered causes. And Hume-ian habit fails to explain such cases.

(d) If belief expresses a vivid idea, how does Hume explain our doubt in a proposition when its truth or falsehood on a par? He may answer by saying that our idea of the existence and the non-existence of its objective reference is not vivid. If we are in doubt whether rain fell yesterday, then our idea of falling idea or not-falling are faint, and thus no belief. Further, Hume's criterion of belief does not work when we have no doubt but probability of an occurrence.

Now, we may provide some criteria for the probability of rain's falling yesterday, namely, clouds or bad weather etc. If these are noticed to occur in most cases when rain falls, then there is probability that rain falls when such conditions occur gain; then the idea of rain's falling is probable not a belief.

<u>Probability is of two kinds</u>: (a) that which depends on frequency such as the high frequency of rain's falling in the example mentioned above, (b)

that which depends on a logical basis. For instance, suppose we are told of the death of only one person among the passengers of an aeroplane; suppose further that all the passengers were three men, then the probability of the death any of them is 1/3, the death of one among two of them is 2/3 Now, Hume's theory of probability fits with the first kind but not with the second.

Physiological Explanation of Induction

We have hitherto discussed Hume's psychological treatment of induction. We now turn to the physiological treatment of induction, that is, explaining it in terms of conditioned reflex provided by Behaviourism. Such theory regards inductive inference as a sort of correlation between a conditioned stimulus and certain reaction, instead of a correlation between two ideas in the mind in Hume's theory. The law of conditioned reflex is the Behaviouristic starting point which may be stated as follows. When an event leads to a certain reaction, the former is a stimulus, the latter a reaction, and if such event frequently occurs together with something, this something is said to be sufficient to give rise to that reaction. This law applies equally to man and animal. The traditional example of the law is Pavlov's experiment of a horse which is found to have more saliva on seeing food; when we condition the appearance of food with ringing a bell, it is found that saliva increases as the bell is heard even if there is no food. Ringing the bell becomes a condition of a certain reaction.

Now, Behaviourism claims that inductive inference could be explained in terms of some form of that law; for example, if (b) being a stimulus leads to certain reaction, and if (a) is frequently found to occur with (b), (a) becomes a conditioned stimulus giving rise to that reaction. Thus, we know the existence of (b) whenever we see (a).

In comment, two points may be stated. First, is the reaction stimulated by (b) what we mean when we say we perceive (b), or we mean that the perception is represented in a psychological element by such physiological reaction? Second, is it possible to explain induction merely as discovering a conditioned stimulus for it? It is the latter question that concerns us.

Induction cannot be explained in terms of stimulus reaction, because we mean by inductive inference either that (b) occurs when (a) does, or that whenever we see (a) we see (b). The former case is a particular one, the latter is so general that goes beyond mere observation and experiment. If, for the sake of argument, physiological explanation fits inductive conclusions in particular cases, it does not fit with general hypothetical inductive statements, that is, if (a) happens, (b) does, For generalisation is not a reaction to stimuli but something new. Further, we do not use induction merely to make clear that (a) is succeeded by (b), but we use it also to prove the existence of the external world. It will be argued in the final chapter that the real ground of our belief in the external world is induction, and that inductive conclusions are not merely frequency of occurrence in the field of perception but they have some novelty differing from all previous reactions. Therefore, inductive inference is to be distinguished from [law-] conditioned reflex.

Notes:

[3]J.S. Mill. A system of logic, p. 255, Longman. London, 1947.

[4]Ibid., P. m/m256. Mill gives 5 canons not four; the author neglects the third being repetitious; his third can[on] on here is the fifth for Mill (Translator).

[5]Mill, A system op. cit., p. 260. This method is given in Mill as the fourth canon. (Tr.).

[6](1) Zaki Naguib, Positivistic Logic, pp. 504-8, (Originally in Arabic). Cairo, 1951.

[7]Zaki Naguib, D. Hume (In Arabic) p. 75 Cairo. 1955.

[8]Zaki Naguib, D. Hume, (in Arabic), Cairo, 1954, pp. 85-6.

[9]Positivistic Logic rejects this proposition even as probable, because any proposition not derived from experience is senseless and in turn not a proposition

at all logically speaking though it may be a proposition syntactically.

[10]Al- Sheneiti, The Philosophy of Hume (in Arabic), P. 182

Induction And Probability

Chapter 1. Calculus of Probability

Introduction

We have already said that induction, in its first stage, is a sort of inference; and we shall show in this part that induction in this stage does not proceed from particular to universal and that inductive inference does not give certainty but the highest degree of probability. Thus induction in its first stage is related to the theory of probability, and it may then be well to begin with the latter. We often talk in ordinary life about probability, for example, when we are asked what is the degree of probability of seeing a piece of coin, thrown at random, on its head, our answer is 1/2. If one of John's ten children is blind, what is the degree of probability of one of them, chosen at random, to be blind? The answer is 1/10; but if we have chosen four of them at random, then the degree of probability that one of his children is blind would be 4/10[?]. We shall discuss three things: (a) this ordinary meaning of probability, and then try to find out the axioms presupposed by the theory, which make possible any arithmetical process; (b) In view of these axioms, we discuss the rules of probability calculus, the rules which determine the ways through which arithmetical processes on probability degrees are made; (c) A logical explanation of our ordinary meaning of probability consistent with those axioms.

Axioms of the theory

We shall use "p/h" to denote the probability of the event p, imposing another event h; we shall take this form as undefined notion. Bertrand Russell summarises the axioms of the theory of probability, acknowledging Professor C.D. Broad's work, as follows[11]:

I. Given P and h, there is only one value of P/h. We can therefore speak of "the probability of p given h".

II. The possible values of P/h all the real numbers from 0 to 1, both included.

III. If h implies p, then P/h = 1; we use "F' to denote certainty.

IV. If h implies not-p, then P/h = 0; we use "O" to denote impossibility.

V. The probability of book p and q given h is the probability of p given h multiplied by the probability of q given p and h, and is also the probability of q given h multiplied by the probability of p given q and h. This is called the "conjunctive" axiom. For example, suppose we want to know the degree of probability of one student in the class to be excellent in both logic and mathematics, we say the degree of probability his excellence in both subject [- matters] is equal to the degree of probability of his excellence in logic multiplied by the probability of the student who is excellent in logic is also excellent in mathematics.

VI. The probability of p and (or) q given h is the probability of P given h plus the probability of q given h minus the probability of both p and q given h. This is called the "disjunctive" axiom. In the previous example, when we want to know the degree of Probability of the excellent student in logic and mathematics in a class we get the degree of probability of his excellence in mathematics plus the degree of his excellence in logic, then subtract from this; the degree of probability of his excellence in these matter as determined by the conjunctive axiom, the product will be the degree of probability of his excellence in one of them. These are the six axioms that the theory of probability presupposes, and so we should give probability a meaning consistent with these axioms, that is, the probability of p given h should have a meaning implying only one value in accordance with axiom 1, giving any value from zero onwards in accordance with axiom II, and requires that the value 1 when h implies P, and the value 0 when h implies not-p in accordance with axioms III and IV, etc.

Rules of the Calculus

Rule of sum in compatible probability: If h is a process which necessarily leads to one of the following results a, b, c, or d, then we have the following four probabilities: a/h, b/h, c/h, d/h. If we want to know the probability of finding a/h [or b/h], we reach it by adding the value of probability a/h and b/h, and this means that the probability of finding a certain result equals the sum of the probabilities of finding each result separately. That is, the probability a/h or b/h = the value of a/h + b/h; that is an application of the disjunctive axiom which says that the value of probability of one of two events a or b = the value of a + the value of b- the value of the whole. Assuming that the happening of both events is improbable in incompatible results, it is time that the probability of the happening of an event equals the sum of the happening of both.

The sum of probabilities in compatible collection is 1. Suppose we have two or more instances and that one of them at most must happen, these instances are regarded as inverse, and such collection is called compatible collection. In throwing a piece of coin, its head and tail are compatible collection, because one of them only must occur; having a pamphlet containing ten pages, opening the first page, or the second .... or the tenth is an instance of all the compatible cases. We may thus maintain that the sum probability of compatible cases is always equal to number 1.

Rule of sum in compatible probabilities: If we have two probable instances a and b, [???] may occur together, and we want to know the probability of a's or b's occurring, it is not possible to determine this by adding the value of a and b, but by subtracting the value of the sum from the value of both instances, in order to arrive at the probability of a or b. we can know the same value in another way, that is, by getting a compatible collection consisting of two inconsistent cases, the occurrence of a or b, and their absence. The value of these two cases equals 1 in accordance with what has already been said in the previous paragraph.

Rule of multiplication in condition[ed] probabilities: If we have two probable cases a and b, the value of the occurrence of b assuming the occurrence of a may be greater than the value of the former's occurrence without the latter. For example, it is probable that a student passes in logic and mathematics examinations, but if we suppose that he has passed in logic then there is greater probability of his success in mathematics on the condition of high mentality shown in his success in logic; and vice versa: if we suppose that he passed in mathematics, the probability of his success in logic is greater. Probability which is affected by another probability is called "condition[ed] probability". If (a) stands for student's success in logic exam, (b) for his success in mathematics, (h) for membership of students, we get: value of probability of both a and b = value of the probability (a/h) + b/(h + a).

Rule of product in independent probabilities: There may be unconditioned probabilities, e.g., the probability that x will pass in logic exam., and y will pass in mathematics. The value of probability of the one is equal to that of the other; such is called "independent probability". If a stands for John's success and b for Smith's, h for studentship, we get a/h = a/(h + b). In such a case, the probability value of both a and b = the probability value a/h x b/h in accordance with "conjunctive axiom".

Principle of inverse probability: The conjunctive axiom tells that if we have two events (p and h), given the conditions of their happening (q), we get : and h/q = (p/q) x [h/(q + p)] The conjunctive axiom entails:

$$p / (q \text{ and } h) = \frac{(p/h) \times q / (p \text{ and } h)}{q / h}$$

That [is] to say, the probability of distinction of some student in math exam provided he satisfies certain circumstances, and supposing he is as distinguished in logic as math provided such circumstances multiplied in the probability of his distinction in logic supposing he is distinguished in math, all is subtracted from the probability of his distinction in logic provided such circumstances.

Such equation is called "inverse probability". By virtue of this principle, the value of probability of gravitational theory after Newton was determined, in such a way the planet Neptune was discovered.

Bags example and Probability calculus: Take the famous example of bags. Suppose we have three bags each of which contains five balls. The first bag contains three white balls, the second contains four white balls, all [five] balls in the third bag are white. Suppose again that we take a bag without knowing which, and draw from it three balls and are found to be all white, then what is the probability that this is the third bag the balls in which are all white? The probability is 2 / 3. This may be explained thus:

$$\frac{1/3 \ \text{x} \ 1}{1/3 \ \text{x} \ 1 + 1/3 \ \text{x} \ 1/10 + 1/3 \ \text{x} \ 4/10} = \frac{1/3}{1/2}$$

Bernoulli's law of large numbers

Let us illustrate this law by the case of tossing a coin. Suppose you tossed a coin n times, and that the proportion of heads in each time is 1/2, then what is the probability of tossing the coin on its head once (m) and on its tail n-m times? Since this may happen in various forms, it is possible to take one of these forms such that m many times and n-m many other times, and then calculate the value probability thus : $(1/2)^m$ x (1 - 1/2) n-m [???]

Further, we have got to know the number of m's in n; this can be shown thus:

$$\frac{n \times (n-1) \times (n-2) \times n- (m-1)}{m \times (m-1) \times 2x\ 1}$$

We can get the value when we get the value of variables.

Now, if the variable 'e' stands for an event, e for its absence, we want to know the number of times in which the event most probably occurs in n times. Suppose that we know the probability value of the occurrence of e at one time and give such value the variable q.

Bernoulli's equations give us the solution. Suppose we give certain number of times in n the variable r, and value probability of the occurrence of an event pe. First we get the fraction [pe (r +1)]/ [pe(r)]. And if we want to know which is larger, nominator or denominator, is it 1 or less or more? We know this when we get the value of this fraction. Let us look at the formula : [(n-r)/(1-r)] x [q/(1+q)] if we want to determine the value of r, we get the relation: pe (r + 1) pe (r), then it is larger than 1. 1 is smaller than [(n-r)/ (1-r)] x [q/ (1+q)]. Then we find that values of r are always smaller than n x pe-(1-pe), that is, than the total of times multiplied in the value probability of the occurrence of event subtracted from the probability of its absence.

For if r is equal to such equation, 1 would be equal to [(n-r)/ (1-r)] x [q/ (1+q)]. When any number of times in which the event occurs in n times is less than the number of times x value probability of the event [-] value probability of its absence, this is called the limit.

Notes:

[11]Russell, B., Human Knowledge, pt. V, ch. 2, p. 363.

Chapter 2. The Interpretation of Probability

The object of the present chapter is to provide the different definitions and interpretations of probability that fit with the axioms and propositions stated in the previous chapter. Let us begin with definitions.

(A) Fundamental Definition

If 'a' stands for the expected instances to occur under certain conditions p, and such instances are incompatible and have equal chances, supposing that x is an event that happens is a number of those instances b, then the probability of the occurrence of x is b/a in relation to p. It is observed that such definition of probability presupposes another definition, i.e., that the relation of the instances consistent with x to the sum of expected instances has equal chances.

But such presupposition is not explicitly stated in the original definition. Thus the definition is vague and incomplete. In other words, probabilities are of two levels. On the first level we have the probability of the values of different ways of an event's occurrence in isolation; when we determine the value of every case and suppose that all cases have equal chances, we move to the second level, i.e. the probability of the events related to some of those possible cases and have equal chances. The original definition applies to the second level only, thus its incompleteness.

In order to make our objection clear, we may look deeply into the meaning of equal chances. We get two interpretations. First, we may explain the equality in probable cases by equality of the value probability. Secondly, we may also explain the equal chances in reference to the conditions under which events could occur, and then p includes all probabilities. Therefore all possible cases in reference to n represent one probability. If we take the latter (all probabilities included in p). interpretation we get rid of the objection to the original definition. But then confront two further problems.

The First problem

The first definition of probability already stated faces two problems, the first of which is that its presuppositions are themselves insufficient to justify the assertion that the degree of probability of b's occurrence, in the example mentioned above is b/a. For why should the probability of occurrence of all forms of an events have equal chances? This problem could be overcome in two ways. First, we suggest to add another presupposition, namely, of the possible form of the occurrence of events are equal, then the values of probability are equal. Secondly, we also suggest to remove any doubt as to the probability of happening, and then we get objectivity; we then say that the value of the occurrence of b in all probable case is .[???]- and this is regarded as objective judgment.

Now, we have two sorts of probability, real probability involving credibility, and mathematical one involving the proportion of cases concomitant with b to all cases. But these sorts of probability different, for the first concerns one single case, while the second concerns a hypothesis. For example, if we throw a particular [pi^???] of coin, the probability of getting its head is 1/2; but we can say on the other hand that the probability of getting the head of a coin in any throw is 1/2. The first sort is real probability while the second is mathematical.

In this connection we oppose a certain view in symbolic logic, namely, the distinction between a proposition and prepositional function. The latter includes a variable such as x is human; 'x' has no meaning and then truth or falsehood cannot apply to it. A propositional function becomes a proposition and is true or false when we give the variable a value such as 'Socrates is a man'. Further, when we have a class included in another class we have a prepositional function not a proposition, e.g. Iraqis are intelligent, that is, it is a hypothetical statement meaning if x is Iraqi, x is intelligent, Now, we oppose the view mathematical probability expresses a prepositional function. Mathematical probability, in our view, expresses a proposition for it is not of the same logical type as the inclusion of a class into another class. Indeed, mathematical probability considers two classes of events but involves a relation between them, and such relation is definite. Thus, it can be true or false.

The Second Problem

The second problem for the first definition of probability, stated above, concerns the equal chances, one of which is supposed to occur. But we want to determine the meaning of this equality supposed between the different occurrences and p. This equality presupposes some relation between each probable occurrence [???] and p, that this relation has degrees, and that those occurrences have equal chances in relation to p, if relations to p are all of one degree, no more or less. Now, what is such relation? It may be a relation of probability e.g. the relation of the appearance of a coin on its head in such degree [???] probability, and since the degree of the probability of each occurrence is indeterminate, it may be equal to the probability of any other occurrence supposed to be probable, but it may be larger or smaller.

But this explanation repeats the first problem, that is, the first definition already presupposes probability. Therefore, we must try to explain the relation which connects p with each probable occurrence without supposing probability in the content of that relation. This means that the relation must be constant and independent of probability and certainty; it must be between two propositions, namely, between p and the occurrence related to it, between the statement that a piece of coin is thrown, and that it is thrown on its head. The relation between these two propositions may be that of necessity, or contradiction, or else mere possibility. The relation of possibility is not here the same as probability because possibility, if taken as probability, is not an objective relation independently of perception. Whereas we mean here by possibility the negative of both necessity and contradiction; and since these latter two are objective, so the former. Thus, the objective relation standing between p and each probable occurrence is that of possibility in the sense that it is neither necessary nor contradictory. But it clear that possibility in this sense cannot explain the equal chances between the probable occurrences in relation to p, for possibility has no degrees of equality or largeness or smallness. So we turn to another definition of probability.

(B) Probability in the Finite Frequency Theory

We now turn to another definition of probability on the ground of which the Finite Frequency Theory of probability was established, in order to see whether it avoided the problems facing the first definition considered in the previous section. The new definition does not speak of the probable occurrences of p; neither does it explain mathematical probability in terms of definite domain of those occurrences. The new definition rather considers two classes of things or events, all members of which really exist, the class of Iraq is and that of intelligent instance. Now, what, is the degree of probability of some individual, randomly taken, to be both Iraqi and intelligent? This degree would be the number of intelligent Iraqis out of all Iraqis. The definition of probability according to this theory may be stated thus. If B and A are two finite classes, then the probability that s, taken at random of B, is also an A is to find out the number of B's that are also A's subtracted from all B's.

Such definition satisfies, in our view, the presuppositions and avoids the problems already discussed. For this definition avoids mention of all probable cases of p, which may, or may not be equal; it considers the number of individuals or particulars belonging to one class, and determines the probability that some member of B is a member of A according to the proportion of frequency of A in B, without supposing the idea of equal or unequal chances. But such definition faces a new objection, namely, that it does not exhaust all the cases included in mathematical probability. Before stating the objection in some detail, we may give some preliminary remarks.

Real and Hypothetical Probabilities

To say that there is a certain degree of probability that an Iraqi is intelligent is not the same as to say that if such a man is Iraqi he may be intelligent in some degree. These are different statements.

The former talks about a real probability and it is possible to turn it into certainty provided we get sufficient data about that individual. Whereas the latter considers a hypothesis, the degree of intelligence in the class of Iraqis, and this involves the certainly true statement that there are intelligent Iraqis.

What is expressed in the first statement may be called real probability, in the second statement hypothetical probability. Now we may add that mathematical probability which the definition aims to explain is a hypothetical, not real, probability, because mathematical probability, being deduced from mathematical axioms, is a necessary statement, while real probability is not, because the latter refers to cases about which we are ignorant.

On the other hand, real probability includes two statements. When it is probable that some individual Iraqi is intelligent by 1/2, we actually give two assertions, first that it is probable in 1/2 that such individual is intelligent; second, that if the degree of our knowledge or ignorance of the circumstances related to intelligence among Iraqis is the same degree related to the intelligence of some individual, then such degree is 1/2. The first statement asserts some probable judgment, while the second is hypothetical only, that it asserts a relation between two terms, thus it is certain not probable.

Does this definition exhaust all probabilities?

We obtain probability if one of the three following conditions are fulfilled. First, if we have two classes of things B and A and there exist members of both, then it is probable that those members of A belong also to B. Secondly, if we have two classes B and A, each of which has members, but we do not know whether there exist common members, then it is probable that some members of A belong also to B. Finally, suppose we are told that there exists someone called Zoroaster who assumed himself a prophet, lived between the tenth and sixth century B.C. To say that it is probable that he really existed is to say that he belongs either to a real or null class of prophets. Let us now examine these cases.

Take up the first case, in which we have two sorts of probability: hypothetical and real probability. The former is expressed by saying that it is probable that some x, being a member of A, is also a member of B. And this sort is determined, provided we know the number of common members. Real probability means that it is probable that x being a member of A is really a member of B. This sort of probability can be determined if two conditions are satisfied. First, there must be definite number of the members of a class B that also belong to another class A including the member x; if we assume that B has ten members, one of whom is x, we must know the number of the members of A that are also B. Secondly, we must include in our definition the axiom that there must be consistency between the number of common members in relation to the class B and the degree of probability that x belongs to A. If these two conditions are fulfilled, the definition applies to real and hypothetical probabilities.

The above consideration may involve contradiction, because when we speak of the real probability of x, we mean that we did not examine whether x is member in both A and B. Thus, when we stipulate our knowledge of the number of members belonging to A and B, we assume examining the status of x, and the fall into contradiction, that is, in order to determine the degree of probability of x belonging to A, we must be sure whether it belongs to A or not. But contradiction disappears provided we can know the number of the members of B that belongs also to A without determining an individual in particular. Further, there may be common members of two classes without determining their definite numbers. We have in this case a hypothetical probability in the sense that it is probable that there may be a member of A that is also B, we have here also real probability in the sense that x is probably a member of A.

Let us turn to the other two conditions of probability statements. In them, there is no hypothetical probability, for this means our knowledge of the number of common members in relation to all Members of A, and we have not such number. Again, our present definition does not apply to real probability, because such definition connects the degree of probability with the degree of frequency, but it does not assume the frequency in the latter two conditions. We may conclude that the definition of probability in the Finite Frequency Theory is in sufficient since it does not exhaust all sorts of probability B.

Russell attempted to defend this definition and its relevance to all sorts of probability on the basis of the principle of induction which justifies the generalisation [and] which applies to unobserved instances. "Suppose I say for example : There is high probability that Zoroaster existed". To substantiate this statement, I shall have to consider, first, what is the alleged evidence in his case, and then to look out for similar evidence which is known to be either veridical or misleading ....We shall have to proceed as follows : 'There is, in the case of Zoroaster, evidence belonging to a certain class A; of all the evidences that belong to this class and can be tested, we find that a proposition p is veridical ; we therefore infer by induction that there is a probability p in favour of the similar evidence in the case of Zoroaster. Thus frequency plus induction covers this use of probability'[12].

We many observe the following points on what Russell said. First, the probability of Zoroaster's existence is real and could not be determined on the frequency theory basis, for frequency and induction lead to a definite ratio of truth, and this we called mathematical induction which alone is insufficient to infer the probability of Zoroaster's real existence. In order to give such probability we have to add the axiom, that the degree of real probability of an event must conform to the frequency of various events belonging to the class of which that event is a member. Such axiom is not presupposed in frequency theory and induction, so Russell's attempt is unsuccessful.

Secondly, the explanation of such real probability as Zoroaster's existence on the basis of frequency theory is unsuccessful also unless there is evidence that the probable is a member in a compatible class.

But in such a class, a member may not occur but it does not occur necessarily; thus the required evidence cannot necessarily be assumed.

Finally, the principle of induction itself depends on probability. For induction which justifies the general conclusion does not rest on probability in the sense of finite frequency, but probability in another sense to which we shall turn.

New Definition of Probability

We offer here a third definition which overcomes the difficulties involved in the two previous definitions.

But it may well first to introduce the concept of indefinite knowledge, that is, knowledge of anything not completely determined or defined. When I say I know that the sun rose or that John is coming now to pay you a visit, then I have determined a piece of knowledge such knowledge is not subject to doubt or probability. But suppose I told you that one of your three intimate friends is coming to visit you now, then I give you indefinite pieces of information which involves vagueness and probability belonging to someone yet unknown. Indefinite knowledge is of two kinds: that which includes incompatible items (two of them cannot simultaneously occur), and that which includes compatible items (when two of them can). And we use indefinite knowledge here to be of the first kind.

Now we have before us four things : (1) Knowing something indefinite in content ; (2) the collection of the items any of which may be the object of knowledge; (3) the number of probabilities which conforms to the number of the items; (4) incompatibility of items. We notice that the degree of the number of probabilities is equal to the given information itself; if this is 1 so is the number of probabilities. Consequently, the probability of each item is a fraction.

Now we come to our new definition of probability: Probability capable of determined value is always one of a class of probabilities represented in indefinite knowledge, its value is always equal to a number of items of indefinite knowledge.

If x stands for any such item, (a) for certainty, (b) for the number of items, then the value of x is a/b. Probability here is neither an objective relation between two events not merely a frequency of a class in another, but an incomplete degree of credibility. This credibility is considered a sort of mathematical probability, by which is meant a deduction from certain axioms. In the example of my knowledge that one of my three intimate friends is coming to see me, if we want to determine the value of x' coming, we find it 1/3.

To examine this definition we discuss the following five points: (a) whether it satisfied the axioms of probability, (b) to overcome any difficulty which it involves, (c) agreement of the definition with the mathematical side of probability, (d) whether our definition explains such cases which the finite frequency theory could not, (e) the additional axioms.

The axioms of the new definition

There are two formulae for the probability a/b. (i) it is the happening of x in the context of the other items of our indefinite knowledge, (ii) it is the various degrees of credibility of the happening of x. If we take a/b according to the first formula, we find it consistent with the six axioms of probability.

The first axiom says that there is only one true value of a/b. The second axiom tells us that all the possible values of a/b are the numbers between zero and 1, and our definition satisfies such axiom because if x does not occur the value is zero but if it only occurs the value is one, and if it occurs with others, the value lies between zero and one. The third axiom says that if b entails a, then a/b = 1. The fourth axiom states if b entails not-a then a/b = zero. Both these axioms are true because when the items of a collection include the member of the probability of which we want to determine, we find a/b = 1 and when such member is absent the probability is zero. The fifth axiom (that of continuity) tells us that the probability of a and c occurring simultaneously in relation to b is the probability of a in relation to b multiplied by that of c in relation [to] a and b; and the value of this probability is consistent with our new definition, not an added assumption. For example, suppose it is probable that some student is excellent in the subject of logic or in that of mathematics or in both. We face here three probabilities each of which is an instance of the probabilities in an indefinite knowledge. On the basis of induction we may suppose two reasons a and b for excellence in logic, and two other reasons c and d for weakness in logic, and two other reasons c and d for weakness and likewise with his status in mathematics. Now we have an indefinite knowledge in both cases. In the first, such knowledge includes a, b, or c or d; in the second we have *a*, *b*, *c* or *d*. The student's excellence in logic is represented in two items a and b, in mathematics represented in *a* and *b*, then the degree of probability in excellence in each subject is 2/4 or 1/2. Whereas his excellence in both subjects is one of the probabilities in a third domain of indefinite knowledge, which can be represented in one of the flowing sixteen cases, a and *a*, a and *b*, a and *c*, a and *d*, b and *a*, b and *b*, b and *c*, b and *d*, c and *a*, c and *b*, c and *c*, c and *d*, d and *a*, d and *b*, d and *c*, d and *d*. We now notice that the probability of the student's excellence in both subjects is 4/16 or 1/4.

The sixth axiom (disjunctive axiom) states that the probability of a or c in relation to b[e] is that of a in relation to b and to that of c in relation to b, and subtracted from the probability of both a and c. And this axiom is consistent with our new definition. In our previous example, we found that the value of each of the two probabilities (excellence in logic and in maths) is 1/2, but the probability of excellence in at least one of the subjects has twelve cases; thus the value of probability of his excellence in one of those subjects is 12/16 or 3/4.

We may now conclude that the first formula of our definition is consistent with all the axioms of probability, without assuming any of them *a priori*.

We now turn to the second formula of the definition expressed by a/b as probability in the sense of degrees of credibility. The second, third and the fourth axioms, aforementioned cannot come in terms with this formula of our definition. For the possible value of a/b do not lie between 0 and 1; rather these latter are among the values of a or b; and this is inconsistent with the second axiom. The third axiom says that if b entails a then a/b = 1 but there is no ground for speaking about entailment here. Yet, it must be noted that the acceptability of axioms of probability is arbitrary because some of them may be needed but it is not necessary to need them all.

Difficulties of our definition

The main difficulty facing our definition lies in determining a definite member among the members of a certain class. Let us introduce the following example. Suppose we have an indefinite knowledge that only one of my three friends will pay me a visit to day (John or Smith or Johnson), then how can I determine the visitor? Here we supposed that the class had three members, but there are alternatives: our class may include Smith and those whose names start with J, or may include Johnson and one of Peter's sons (assuming that John and Smith are his sons). If we take the first alternative, then the probability of Smith's coming is taken then such probability is 1/2,in the third we find that the probability of Johnson's coming is 1/2.

Difficulties are enormous if we take the first alternative, For if we know that John has four costumes (a, b, c and d), we can then say that we have six members, consequently, the probability of John's coming is 4/6. We go into absurdities if we suppose that the probability of one's coming increases for the one whole has more suits.

We can offer two ways of overcoming this difficulty, namely, (1) when one of the members of a class is divisible other members must be so, or else we must ignore divisibility in all members; (2) if one member is divisible but the rest is not, then we should not neglect this process in the former.

The new definition and the calculus

We may well notice that our new definition completely explains the mathematical side of probability. It has already been shown that the axioms of conjunction and disjunction are consistent with our definition. And since the sums and products of probabilities rest on these two axioms, we conclude that the new definition explains all processes of addition and multiplication. In what follows, we take three cases of mathematical probability and see whether they are consistent with the new definition.

The new definition and inverse probability

We want first to discuss the principle of inverse probability in the light of the new definition of probability.

Suppose we draw a straight line and divide it in two parts a and b; suppose also we wish to fire a bullet on a certain point on the line but we do not know whether the point is on a or b, and we found that we fired successfully. Now what is the degree of probability that the point is on a? It will be 9/10 according to inverse probability, and such cases which apply to this principle involve an indefinite knowledge. In saying that the probability of throwing the bullet successfully on the meant point on a is 3/4, we mean that, by induction, we succeed after trying three throws out of four. Now when we fire on the meant point we find we have 16 probabilities six of which are improbable assuming we have already succeeded. The result will be that the degree of probability is 9 /10.

The definition and the Bags - example

In the Bags - example, it is supposed that we have three bags, of which the first contains three white balls out of five, the second contains four white balls and a black one, the third bag contains five white balls. Suppose we took one of the bags randomly and drew from it three balls and found all of them white, then what is the probability that such bag is the third one? Here we have indefinite knowledge and need to determine it; we have indefinite knowledge that the three white balls are either from the first or second or third bag. We have only one chance if what we drew is from the first bag, four chances if from the second bag, ten chances if what we drew is from the third. Thus there is indefinite knowledge involving fifteen chances, each of which is considered a case of such knowledge. If the three white balls are drawn from the third bag then the degree of probability is 10/15 or 2/3 and this is exactly what Laplace calculated in the Bags - example, for he determined this probability by (m+1)/ (n+1) in stands for the number drawn, n all the balls, and this would be 2/3. Now, we may ask, what is the probability that the next ball to be drawn is white from the third bag? In this bag we have two balls left, thus we have two probabilities, when multiplied in our fifteen cases we get thirty cases in our indefinite knowledge.

On the other hand, the probability that the next ball to be drawn is black has 24 cases, thus the probability is 24/30 or 4/5 ; and this is what Laplace found in his equation (m+1)/ (n+1) or (3+1)/ (3+2).

Our definition and Bernoulli's law

Bernoulli's law of large numbers states that if, on each of a number of occasions, the chance of a certain event occurring is p, then, given any two numbers a and b, however small, the chance that, from a certain number of occasions onward, the proportion of occasions on which the event occurs will ever differ from p by more than b, is less than a. Let us illustrate this by two examples.

The first example

It is that of tossing a coin. We suppose that heads and tails are equally probable, i.e., 1/2. Suppose we tried tossing the coin four times, we would have indefinite knowledge of the following occasions : (1) we get heads in all times, (2) heads do not occur even once, (3) we get head once, (4) we get it twice, (5) we get it three times.

The first occasion has only one chance, the second occasion has also one chance, the third has four chances, the fourth six chances, and the fifth has four chances. Consequently, we have indefinite knowledge of sixteen cases, one of which may occur. We can determine the degree of probability that any case may occur independently of the occurrence or the non-occurrence of another case. Such degree is 1/2 because if we randomly choose any of the four occasions and observe the times of the occurrences on such occasion, we find that it is 8/16 or 1/2. We can also determine the occasion, among the five we have, which may gain the highest probability; we shall find that the fourth occasion is such, namely, that the head of a coin appears twice, and this is 1/2. Bernoulli's law of large numbers proves that the form which involves a ratio of occurrence corresponding to its probability will increase.

The second example

Bernoulli's law proves that, provided that the probability of an event is 2/3, on many occasions we may be almost certain that the degree of occurrence of such event is 2/3. It may be asked whether this law could be explained in terms of indefinite knowledge, and we claim that it could.

For since we talk about probability the degree of which can be determined, and if we suppose that the probability of an event is 2/3, this means that this degree is determined according to indefinite knowledge. Thus in the example of tossing a coin many times, we have two sorts of indefinite knowledge: (a) indefinite knowledge which determines the probability of seeing the coin on its head is 2/3; (b) indefinite knowledge which includes all the alternative cases in which the event may appear. When we mix these two sorts we get a third in which all alternatives are equally probable.

Completeness of our definition

The definition of probability on the Finite Frequency theory is incomplete and involves gaps. For suppose we look into statistic results about the frequency of cancer among smokers, and we are not sure whether it is 1/4 or 1/5 owing to difficulty of reading, then the probability here is in the frequency not in the number of smokers, but such probability is not included in the account of that theory.

But such is satisfied in our definition according to indefinite knowledge. That is, the frequency is either 1/4 or 1/5 and thus the ratio is 1/2. There is one exception to our application, namely, complete doubt as to the major principles and axioms such as non-contradiction. This sort of doubt is beyond our definition to include.

New axioms

New axioms of our new definition of probability may be introduced. If we have two kinds of indefinite knowledge, each of which contains many value probabilities, and there can be no incompatibility between them, then we can determine the value probability of one knowledge independently of the other. But if the values of the two kinds of knowledge are incompatible, we can multiply the number of in items both kinds and obtain greater indefinite knowledge. By virtue of multiplication, the value of an item differs in such greater knowledge from its value in its special kind. Suppose we have a coin and another piece having six sides numbered from one to six and tossed both, we have two kinds of indefinite knowledge: first, knowledge that the coin may be on its head or tail; secondly, knowledge that the six - faces piece may fall on a certain face. This means that the probability of the coin's appearance on its head is 1/2, and that of the other piece is 1/6.

Now, if we knew that, for certain reasons, the head is concomitant with a certain number in the other piece, then the degree of probability of the coin's appearance on its head will be less than 1/2. For we have to multiply the probabilities of the items of one of indefinite knowledge in the items of the other; then we get a new indefinite knowledge consisting of seven probabilities : (i) head with number 1, (2) head with number 2, (3) head with number 3, (4) head with number 4, (5) head with number 5, (6) head with number 6, (7) tail with number 6. Consequently, by multiplication the value of the appearance of the Coin's head is 1 / 7 and the value of the other piece's appearance on number 5 is 2 / 7. When we have two sorts of indefinite knowledge which can constitute a third sort by virtue of multiplication, the value probabilities will differ in the third sort from those in the former this we may call the multiplication axiom in indefinite knowledge.

But we need another new axiom. For in many cases in which we have two sorts of indefinite knowledge, and some items in the one are incompatible with some items in the other, we notice that the value probabilities are determined within one sort without the other. In such cases we have no need of the multiplication axiom, but another axiom which we may call dominance axiom. Let us make this axiom clear first by example.

Suppose we have indefinite knowledge that some person in the hospital (c) is dead, and we know also that there are ten sick persons in c; thus the probability that anyone of them is dead is 1/10. But take the following case. Suppose there is a sick man, besides the ten persons we know of, but we do not know whether he went to the hospital (c) or another (b) in which nobody died; and suppose that his entry in either hospitals has equal probability. This means that there is a second indefinite knowledge that the eleventh person is in (c) or (b), and that the probability of his being in either hospitals is 1 / 2. In this case, the eleventh person stands in the domain of the first indefinite knowledge, because since it is probable that he is one of the clients in (c), it is probable that he is the one we know about his death.

Hence, the probability that the dead man is in (c) is 1/10. We notice that the probability that the eleventh person is in (b), involved in the second indefinite knowledge, and the probability that he is in (c), involved in the first indefinite knowledge, cannot be both true. We now come to state the second new axiom presupposed in our definition of probability namely, dominance axiom: If there are two probability value derived from two kinds of indefinite knowledge, and if one of these value affirms, and the other denies some event, and the one includes the other, we call the former dominant over the other.

Ground of Dominance Axiom

There are two grounds which justify the dominance axiom. The first ground is that we should acquire a knowledge that what is to be known in the first indefinite knowledge possesses a quality necessary to one item in it but not necessarily belonging to other items in the same piece of knowledge; in this case, any probability incompatible with those other items dominates the probability compatible with the first item. For example, we might know in an indefinite manner that John or Smith is in the room and we know by testimony that the person there is white, and we know that Smith is white but we do not know John's colour. Then whiteness is the quality we know of the object of our indefinite knowledge and that it is necessarily possessed by Smith and has no connection with John's colour. Now any factor weakens the probability that John is white dominates the probability that John is in the room, What is known is the presence of a white man in the room; when we become sure that he is white we get higher probability.

In other words, the first ground is that if a certain quality is attributed to any item of a group of items is equally probable, there can be no dominance. In the previous example, if we know that Smith only is the person that is white then we are certain that it is he that is present in the room and thus we do not have indefinite knowledge.

The second ground of dominance axiom is that when we have indefinite knowledge about something, and that the object of knowledge may have some quality not necessarily possessed by any item, then any probability that such quality is, or is not, attributed to an item dominates the previous probability we have. For example, suppose we know that there is a white man in the room and we are told that he is either John or Smith, and we have no clear idea of the colour of both. Whiteness here is a quality that is not necessarily possessed of either. Then if whiteness is equally probable for both then the probability that either of them is in the room is 1/2. Now if we have knowledge that decreases the probability that John is white such knowledge dominates our previous probability.

Categorical and Hypothetical indefinite knowledge

Statements are of two sorts: categorical and hypothetical; the former attributes a predicate to a subject, and expresses a fact, while the latter expresses a relation between two facts by virtue of the fulfilment of a certain condition. We may apply such classification to indefinite knowledge and say that the latter may be categorical or hypothetical. An example of the former is the knowledge that your brother will visit you; example of the latter is that your brother will visit you in the period of the next ten days if he is not ill. As any categorical indefinite knowledge includes a number of items as members, so hypothetical indefinite knowledge includes a number of hypothetical statements each of which may be considered a member of the original statement; and each is probable. In the previous example we have ten hypothetical probable statements :

1) X will visit his brother tomorrow if he is not ill.

2) X will visit his brother the day after if he is not ill.

....

10) X will visit his brother in the tenth day if he is not ill.

The probability of any of those statements equals 1/10. Hence we have an important point, namely, that if the condition is a probable fact and if this fact has ten chances, some of which have the least probability, then we get a value probability inconsistent with the occurrence of such fact. Let us make this example clear. The condition is a probable occurrence that a person is not ill, and we have ten conditional statements turning on this occurrence.

Suppose we know that the person in question did not visit his brother in the first nine days and we know nothing about the tenth day then all probable conditional statements are inconsistent with the condition, therefore that person is ill.

Now, when the consequent of the conditional statement is false, then the antecedent or condition is absent, thus, we want to determine the probability of x's illness with 9/10 in the first nine days, but we know nothing of his visit in the tenth day, then we can say that our conditional indefinite knowledge gives the probability of his illness with 9/10. In consequence, we may formulate our axiom thus: every conditional indefinite knowledge includes a number of probable conditional statements, all having one condition in common but differing in consequent ; then this knowledge denies the existence of the condition with a probability equal to the probability of the original statement.

Conditional knowledge that is real

Conditional indefinite knowledge are of two kinds, (a) knowledge of the consequent which may be real, but being ignorant of it we formulate a conditional statement that gives us alternatives one of which is realizable in reality and the rest are probable. For example, I may indefinitely know that if I take a certain drug I may suffer one of three sorts of pain, in this case I can consult an expect to tell me which sort is to happen. (b) Conditional knowledge which involves a number of alternatives, none of them is real. For example, if we have a bag containing a number balls, and at least one of which is black, and ask which one is so? We have then indefinite knowledge that one of the balls is black.

These two kinds of conditional indefinite knowledge are substantially different; the kind which indicates the non-existence of the consequent in reality involves that there is no contradiction for the consequent to exist but that there is no empirical ground for its real existence. Whereas the kind of knowledge which gives empirical information implies only some sort of doubt such that if I have enough knowledge I could have obtained definite knowledge without any doubt. This main difference suggests that indefinite conditional knowledge which involves no empirical information cannot be taken 'a ground for the determination of any probability, where as the other kind of conditional knowledge can be taken a ground for such determination. Therefore, we can discover a mistake in applying the theory of probability in certain cases.

For example, if there is a bag containing ten balls numbered from 1 to 10, and we know nothing of their colour; suppose we draw the balls from 1 to 9 and we noticed they are all white. Can we apply the theory of probability and say that there is a probability that the tenth ball is white on the ground of our indefinite knowledge that if the bag contains a black hall it would be the first or the second ... or the tenth? Such conditional indefinite knowledge includes ten probable statements all of which have a condition in common, namely that the bag contains one black ball. We know that the consequent in the first nine conditional statements is not empirically verified, since we know that the nine balls are white. This means that those statements prove the absence of the antecedent.

Such application of probability theory, we argue, is false because it determines the probability of the tenth ball being white on the basis of our conditional indefinite knowledge which involves the unreality of the consequent; and this basis does not justify the determination of real probability.

Recapitulation

We have hitherto studied and discussed the theories of probability, and offered a new definition of probability from which the following results may be uncovered. First, probability always depends on indefinite knowledge, and the value probability of any statement is determined by the ratio of the number of cases involved in this statement to the total number concerned. Second, a theory of probability based on our new definition has, for its ground, five postulates (a) the objects of indefinite knowledge have equal chances; (b) if some items of indefinite knowledge may be classified further while other items do not, then the division involved in the former is either original or peripheral; if it is original then each item is one of our indefinite knowledge, while if it is peripheral, then the item is the only member of such knowledge; (c) if we have two kinds of indefinite knowledge having distinct probabilities, one of which is consistent with a certain statement, while the other is inconsistent with this statement, such that one of those probabilities denies the statement while the other does not, then the former dominates (or exhausts) the latter; (d) when conditional indefinite knowledge involves the unreality of the consequent, it cannot be taken as ground for a probability of the consequent. Finally, if we have two kinds of indefinite knowledge, the value probability in the one is inconsistent with that in the other, we must multiply the members of the first kind by the members of the other kind, and then obtain a wider knowledge.

Notes:

[12]Russell, B., Human Knowledge, pt. V, ch. 2, p. 363.

Chapter 3. The Deductive Phase Of Induction

Introduction:

Induction has two phases: deductive and subjective, and we have still been concerned with the former. By deductive phase here is meant that inductive inference aims at generalisations, and in order that these be effected, induction finds help in the study of probability. But we get the highest degree of probability of our generalisation in a deductive manner, that is, deduction from certain axioms and postulates. Therefore the degree of probability of inductive inference depends on such axioms and postulates. In this chapter, we shall consider induction as enriching the probability of generalisations, depending on the postulates of the theory of probability without giving any extra postulates for induction itself.

We shall explain our new approach to induction and probability in relation to a certain form of causal principle. When we say that motion is the cause of heat, or a metal exposed to heat is the cause of its extension, we intend to confirm such generalisation by induction.

Causality

Causality is a relation between two terms such that if one of them occurs the other does necessarily according to rationalistic theory, while such relation according to empiricism expresses constant or uniform conjunction which involves no necessity. The form definition of causality designates a necessary relation between two meanings or classes of events. When we say that motion is the cause of heat, we mean that any particular occurrence of motion is necessarily succeeded by a particular occurrence of heat. Whereas causality, on empiricist lines, is a relation of uniform conjunction involving no necessity.

But the denial of necessity involves complete chance. For instance, heat precedes boiling is as chance occurrence as that falling rain precedes visiting my friend, with the difference that the former involves chance uniformly while the latter involves chance rarely. Thus, when succession of events is a uniform chance, it is a relation between two particular events not between two types of events.

We may distinguish positive and negative causality, namely, that when an event occurs, another follows, and an event does not occur because its precedent condition does not. It is to be remarked that, for rationalism, negative causality involves the impossibility of complete chance, that is, the occurrence of some event is impossible without the occurrence of its cause. But positive causality does not involve the impossibility of complete chance, for that (a) is succeeded by (b) is not inconsistent with the occurrence of (b) without (a). We may now conclude (1) that positive causality does not deny complete chance from the rationalistic point of view. (2) that positive causality involves complete chance from the empiricistic point of view, (3) that negative causality, for rationalism, implies that complete chance is impossible.

Now, we shall, in what follows, give four applications to our conception of *a priori* causality so as to clarify the deductive phase of induction. In the first, we claim that there is no *a priori* ground to deny positive causality on rationalistic lines, and that absolute chance is impossible. Second, we claim that there is no *a priori* ground for believing or disbelieving in negative causality, that is, throwing doubt in absolute chance. Thirdly, we shall defend the view that the belief in absolute chance is consistent with the belief in positive causality. Finally, there is no *a priori* ground for denying positive causality on rationalistic lines, but we may claim at the same time that causality stands, as uniform conjunction.

First Application

As rationalism claims, we assume that there is no *a priori* ground for denying necessary connection between cause and effect, and that there is ground for denying absolute chance. Take this inductive statement: "all A is succeeded by B", and you find before you three probable formulae:

(1) the generalisation: all A is succeeded by B.

(2) A is the cause of B in view of empirical data.

(3) A is the cause of B independently of experience, and the last is also probable so long as we have no ground for denying it.

We notice that the first two formulae are one, while the third is distinct. Inductive inference, in our interpretation, proves the causal principle and thus confirms generalisation in the specified way as we shall presently see.

In order to deal with inductive inference, we have recourse to the concept of indefinite knowledge, and since we assumed *a priori* the impossibility of absolute chance, we mean that b must have a cause. Suppose it is probable that the cause of b is either a or c, and by experiment we find that a is concomitant with b, we now have two cases, either that c does not occur or that c could occur. In the former, we conclude that a is the cause of b and we need not use indefinite knowledge, because we reached the causal relation between (a) and (b) priori and deductively not through induction. But in the case where (c) could occur in conjunction with (b) (but actually did not), we may say that (a) is not definitely the cause of (b), and then their conjunction could be explained in terms of relative chance. Hence we need to introduce indefinite knowledge to judge the probability that (a) is the cause of (b) in the following way:

(1) (c) did not occur in both experiments,
(2) (c) occurred in the first experiment only,
(3) in the second only,
(4) it did occur in both.

It is remarked that first three cases show that (a) is cause of (b) while the last is indifferent as to confirm or deny such causality. This means that we have three probability value in favour of affirming the causality of (a) to (b), therefore the probability that (a) is cause of (b) in both experiments is 3.5-4 = 7/8, and after three experiments 15/16, and the probability increases when we make more experiments. Such indefinite knowledge may be called *a posteriori*, since it enlarges the causal principle through induction.

Rule of multiplication

In addition to *a posteriori* indefinite knowledge, there is, we suggest, an *a priori* indefinite knowledge, and the latter is already conceived before inductive process. If we suppose that (b) has either or (c) as cause, this means that such knowledge includes two items only: (a) and (c); this knowledge determines the probability that (a) is the cause of (b); in this case, the value would be 1/2, and the denial of it would also be 1/2. Again, after two successful experiments, we get two pieces of indefinite knowledge, the *a priori* and the *a posteriori*; former gives the value 7/8, while the latter gives 1/2 as a determination of causal relation.

Now, we can apply the rule of multiplication to those two pieces of indefinite knowledge and a third indefinite knowledge issues. After having two successful experiments, we can have eight probabilities, four within the *a posteriori* knowledge multiplied in two within the *a priori* knowledge.

(1) The assumption that (a) is cause of (b) and (c) coexists in both experiments

(2) That (a) is cause of (b), and (c) occurs only in the one experiment,

(3) that (a) is cause of (b), and (c) occurs only in the second experiment,

(4) that (a) is cause of (b), and (c) disappears in both experiments..

(5) that (c) is cause of (b), and (a) occurs in both experiments,

(6) that (c) cause of (b), and occurs only in the first experiment,

(7) that (c) is cause of (b), and (c) occurs only in the second experiment,

(8) that (c) is cause of (b), and (c) disappears in both experiments.

It is noticed that the latter three cases never occur since they involve that (b) occurs without any cause. Thus remain the five cases, which constitute the new indefinite knowledge, and since four out of these five cases involve that (a) is cause of (b), then the value probability here is 4/5 instead of 7/8. Suppose we have made three successful experiments, and that the *a priori* indefinite knowledge has two items only, then the probability that a is cause of b, according to the *a posteriori* knowledge, would be 15/16, and according to the multiplied knowledge, 8/9.

Application of Dominance Axiom

The rule of multiplication, discussed above, apply only to the domain of probabilities which have equivalent values, but does not apply to those values dominating other values. And it is observed that the value probabilities in the *a priori* indefinite knowledge dominate those in the *a posteriori* knowledge. Let us clarify this statement.

The object of *a priori* indefinite knowledge is universal, for instance, something (b) must have a cause which is still indeterminate, it may be (a) or (c). But this universal knowledge, in case two successful experiment were made, denotes the occurrence of something in both experiments. The first indefinite knowledge is that the cause of (b) occurs in both experiments, while the second denies with high probability, that (c) occurs in both experiments, and this negative value provided by *a posteriori* knowledge denies the occurrence of (c), thus any value denying the occurrence of (c) in both experiments denies its occurrence *a priori*. Then such denial dominates the value that (c) is the cause *a priori*.

What has just been said shows clearly that the value of the probability that (a) is the cause of (b) after any number of successful experiments is determined by *a posteriori* knowledge only, not by the third indefinite knowledge produced by multiplication. And the value of probability that (a) is cause of (b), on the ground of dominance axiom is larger than its value on the ground of multiplication supposed by the principle of inverse probability.

Dominance and the problem of a priori probability

By means of dominance axiom we can solve one of the problems which face the application of the theory of probability to inductive inference. Such problem arises from applying the rule of multiplication and the principle of inverse probability, and this application involves the incompatibility between the *a priori* and *a posteriori* indefinite knowledge toward the determination of causality.

A posteriori knowledge may determine that (a) is cause of (b), while *a priori* knowledge does not. Such incompatibility involves that the probability that a causes b decreases according to multiplication. And it is clear that *a priori* knowledge, being prior to induction, does not give determinate causes, but should suppose a great number of them, and then the number of causes suggested here would exceed what *a posteriori* knowledge shows.

This problem is solved by dominance axiom, which shows that the probability that a is not cause of b (which *a priori* knowledge suggests) is dominated by the value probability which *a posteriori* knowledge gives us, and not incompatible with it.

Second Application

We shall now assume that there is no *a priori* basis for denying causal relation between events, and for the impossibility of absolute chance. That is, it is probable that b may have a cause, and that b may at the same time may have no cause at all. Thus, for the sake of argument, we are not permitted to conclude that a is the cause of b merely from their concomitance, and thus b may have occurred by absolute chance.

Now, suppose that a is probably the cause of b in order to reject the probability of absolute chance, and hence to suggest *a priori* the causal relation between them. We shall understand inductive inference in such a way that we can apply the probability theory to the impossibility of absolute chance, as a consequence of many successful experiments, thus we get a high degree of credibility that absolute chance is impossible.

By absolute chance we mean the absence of causality, i.e., the absence of cause is a cause of the absence of effect. We could obtain a hypothetical indefinite knowledge as a result of observing that in all cases when a is absent, b is so. But if the absence of causality is not constant and regular, it is probable that the absence of effect is preceded by the absence of cause. If we observe two cases in which the absence of effect is concomitant with the absence of cause, it would be necessary that the absence of cause is connected with the absence of effect; if not, it would not be known that the absence of effect is connected with the absence of cause. In this supposition we have four probabilities.

The absence of effect does not occur in both cases

The absence of effect does not occur in the first case only. The absence of effect does not occur in the second case only. The absence of effect does occur in both.

These four probabilities express four hypothetical probable statements, all of which have in common one condition, namely, assuming the causal principle. The consequent in the first three statements is false, then the only way to make them true is to assume that the antecedent is false. Therefore the absence of effect as a result of the absence of cause is equivalent to the impossibility of absolute chance.

The impossibility of absolute chance absorbs all the probable values involved in any hypothetical indefinite knowledge, except the value of the statement "unless the absence of cause is a cause of the absence of effect, this would occur in all cases, because all these values deny the condition or antecedent, thus proves the impossibility of absolute chance". And when we compare this hypothetical knowledge with *a priori* knowledge of the impossibility of chance, we do not find dominance of one on the other thus we may apply the rule of multiplication to both sorts of knowledge. And since there is no a priori ground of preferring the possibility of absolute chance to it impossibility, it may be that the value of *a priori* probability of impossibility of chance is 1/2. By multiplying both sorts of knowledge, we shall find that the value probability of impossibility of chance is less than the value determined by hypothetical knowledge only.

Now, if it becomes reasonable that absolute chance is impossible with a greater value it becomes more probable that a is cause of b, for the impossibility of chance implies that a causes b; and if we suppose that b may probably have another cause than a, such as c or d, it is possible, in order to argue against such probability, to take the same way of explaining inductive inference stated in the previous application. We have now argued that it is possible to use the probability theory in order to argue against absolute chance, on the ground of a hypothetical indefinite knowledge.

Third Application

We shall here suppose that there is no *a priori* ground for denying causal relation between two given events such as a and b; we shall also suppose that there is *a priori* ground for the possibility of absolute chance. Such supposition of *a priori* ground does not enable us to strengthen the impossibility of absolute chance as we have seen in the second application. In the third application, our problem is not that the cause of b is not a but may be c or d. Our problem is the probability that b has occurred by mere chance. Now suppose that although the only probable cause of b is a, we could not observe any concomitance between them in varied experiments. If a is a cause of b, it is necessarily connected with b in all relevant experiments. Whereas if we suppose that a is not cause of b, it is not necessary to be concomitant with it; in such a case we have four probabilities, expressed in four hypothetical statements:

(1) Assuming the denial of causality between a an b, it is probable that b does not occur in the two experiments; or (2) b does not occur in the first experiment only; or (3) b does not occur in the second experiment only; or (4) b does occur in both experiments. All these statements are probable, though the condition or antecedent is false in the

first three of them, thus these three statements affirm the causal relation between a and b.

Multiplication or dominance

We may remark that, besides hypothetical indefinite knowledge, there is *a priori* indefinite knowledge, on the ground of which we can determine the *a priori* probability that a is cause of b. Supposing that if there is a cause of b, this cause is no other than a. Thus, there is an integral collection of cases which constitutes the causal relation, and its absence, between a and b; thus the value of each of both probability *a priori* is 1/2.

When we compare the *a priori* indefinite knowledge to the *a posteriori* knowledge, on the ground of which we determine the probability that a is cause of b, we find that the value of the probability that a causes b *a posteriori* does not dominate the *a priori* determination of this causality, because *a posteriori* hypothetical knowledge here does not refute anything involved in *a priori* knowledge; on the contrary, the farmer confirms one horn of the latter in higher probability.

In consequence, these two kinds of knowledge have [o???] to be multiplied, and multiplication affects the value probability of causality given *a posteriori*. By multiplication we get, after making two relevant experiments, five cases, for the fourth hypothetical statement is consistent both with the supposition that a causes b and its denial, whereas each of the other three statements expresses only one case, for it is consistent with the supposition of causality. Thus the value probability that a causes b is 4/5 instead of [-^-^-???]

What has been proposed is based on the supposition that a is the only event that can cause b. But if there are many events other than a, we may modify our hypothetical knowledge in the following way: "If none of the things, concomitant with b in successful experiments, is a cause of b, either ... or ....". Such modification helps to give higher probability that some of the events regularly concomitant with b is its cause.

Secondly, the probability of empirical causality does not exceed 1/2 through successful experiments, because any experiment involving the conjunction of a and b does not alter such probability except in case of decreasing the factor of multiplication. Suppose the kind (a) has ten individuals, then the value of *a priori* probability that a causes b is a result of ten processes of multiplication of the ten individuals of a by b; and when we observe the conjunction of the first individual of a and b, for example, we can dispense with one of those processes; and this means that if we observed the conjunction between nine individuals of a with b, the value would be 1/2.

Thirdly, *a posteriori* knowledge, cannot be a ground of increasing the probability that a causes b, if we deny rational *a priori* causality right from the start. For such denial is equivalent to absolute chance.

Hypothetical Knowledge And Empirical Causality

There may be a hypothetical knowledge which helps to increase the probability of causal law, even if we refused rational causality. Take the example of a bag containing a number of balls. If the bag k contains ten white balls, we may ask whether it has at least one black ball, and then we want to determine which one of the ten balls it is. If we suppose that there is a black ball in our bag, it is probable that it is the ball (1) or the ball (2) ... or the ball (10). That is, when we produce a hypothetical statement the antecedent of which supposes there being a black ball, we face ten probabilities in the consequent, although as a matter of fact there is no black ball.

Similarly, if we suppose that there is a causal relation between two kinds a and b, that the kind a contains 10 individuals, then to say that a is uniformly and regularly conjoined to b is to express 10 conjunctions. Now, suppose we observed [a^1, a^2...a^5][actually these the numbers with these letters should be in subscript, shouldn't they be?] and found that b is conjoined them all, we therefore conclude that b is conjoined to a^1...a^5, but we doubt whether b is conjoined to the other five cases.

But when we doubt causal laws, that is, at least one individual of a is not conjoined to b, then we may ask which one is it? We have no way to know that this supposed individual is one of the unobserved five cases, because it may be that all the ten cases are factually conjoined to b. Thus, we may obtain a hypothetical indefinite knowledge including ten hypothetical probable statements. Such knowledge involves that if there is in the kind a at least one individual not conjoined with b, then it is either a^1 or a^2... a^{10}. The antecedent in all such statements would be the supposition that at least one individual of a is not concomitant with b. We can know that the consequent in five of those statements in variant, and this means that those first five statements are modus tolless [???tollens] (denying antecedent) i.e., denying that there is an a which is not conjoined with b, and this affirms causal law. Thus, in our conditional knowledge, we have five hypothetical statements which favour causal law, while the other five statements are indifferent to the law. But the more there is conjunction between a and b, the more we have of hypothetical statements which affirm this law.

But the role performed by conditional knowledge is no ground for increasing the probability of causal relation to a reasonable degree for two reasons. First, we have already distinguished, within conditional indefinite knowledge between knowledge whose antecedent is factually determined and knowledge whose antecedent is not so. And the role performed by conditional knowledge in increasing the probability of causality lies in the latter knowledge. Secondly, even if we ignore the above distinction within conditional indefinite [???].

Fourth Application

Whereas we considered the previous applications starting from supposing no *a priori* ground of causal relation between a and b, the present application assumes *a priori* ground of refusing such relation. That is, inductive inference involves causality on empirical lines not on rationalistic lines, which means mere conjunction.

Causality, empirically considered, involves not a relation between two events but various relations among many things the relation of the particular (1) of the kind (a) to the particular (1) of the kind (b), and the relation of the particular (2) of the kind (a) to the particular (2) of the kind (b) and so on. Therefore, the causal relation between a and b is a multiple relation between particulars of a and those of b, and such relations express relative chance.

Now, the main difference between causality (in the sense of uniform chance) and causality (in the sense of rational necessity) is that in the former it involves a collection of independent relations, and in the latter it is one relation between the individuals of one kind and individuals of the other kind. In consequence, we may conclude the following points. First, since causality considered empirically, expresses a collection of independent relations equivalent to the number of particulars of kinds a and b, the value of its *a priori* probability is the value of the probability that a particular of a is enjoined to a particular of b, multiplied by the value that another particular of a conjoined with another of b, and so on. So the value reaches a fraction equal nearly zero; whereas causality rationally considered, being one single relation between two events, its *a priori* probability takes the value 1/2 knowledge this is still not to be taken as a ground of causal laws [??? sentence correction ???].

That is to say, any conditional knowledge contains a number of statements equal to the number of individuals included in the kind concerned. If we suppose an individual in the kind (a) not conjoined with b, then this individual is either a^1 or a^2... or a^n, than the number of the consequent is as much the number of individuals in that kind. On the other hand, if conditional statements affirm causal laws their number is equal to the number of individuals under examination or experiment. It follows that the more the individuals of a kind are examined, the minimal value we get for the probability of causal law.

As concerns the deductive phase of inductive inference and its justification, we have reached the following important points. First, the deductive phase is the first step of inductive inference and is a reasonable application of the theory of probability in the sense given in the course of this chapter. Thus induction does not presuppose any postulate except postulates of probability itself.

Secondly, such deductive phase does not assume *a priori* justification for denying causal relations on rationalistic lines. Now, the denial of such causality cannot explain inductive inference. Finally, induction is consistent with the impossibility of absolute chance.

Chapter 4. Modern Theories of Probability

We have explained in the previous chapter the deductive phase of induction in detail and argued that such phase needs presuppose nothing except the axioms of the theory of probability in the way [??? we ???] interpreted it. Now, the pioneers of the theory of probability seem to have taken already the same course as we did, with the difference that they defended the deductive phase of induction even without presupposing anything concerning causality.

Laplace is one such pioneer. In what follows we shall discuss Laplace's position and then compare it with ours. To begin with, we are reminded of the example of bags hitherto mentioned. If we have three bags a, c and d before us, each of which contains five balls, and that the bag a has three white balls, the bag c has four white ones, and all the five balls in the bag d are white.

Suppose we picked out one of these bags at random and drew three balls from it and found them white. Now, the probability that such bag is d is more likely, for its *a priori* value (before drawing the three balls) is 1/3, and its *a posteriori* value is 10/15. There is only one probability that such bag is a since this has only three white balls, and four probabilities that such bag is c, and ten probabilities that is the bag d. Thus we have fifteen probabilities which we have indefinite knowledge that one of them would be fulfilled. Ten of these probabilities concern the bag d, consequently, the *a posteriori* probability is 10/15, i.e. 2/3.

The probability that the next ball to draw is white is 4/5, because there remains in the bag two balls after drawing three ones. Since we probably draw one of them, then we have two probabilities which if multiplied by the fifteen forms referred to, give us thirty forms. These constitute a whole set of indefinite knowledge, twenty four of which involve that the next ball will be white; thus the value would be 24/30 = 4/5. If n denotes the balls drawn, n the whole number of balls, Laplace arrived at the following two equations:

(1) the chance that all the balls in the bag are white = (m+1)/(n+1)

(2) the chance that the next ball to be drawn is white = (m+1)/(m+1)???
And that is true and consistent with our position.

But Laplace wanted to generalise those values within one bag (n) containing five balls, and determine whether it has 3 or 4 or 5 white balls. We get three probabilities: (i) n may be similar to a, i.e., that the bag has three white balls only; (ii) n may be similar to c, i.e., that the bag has four white balls; (iii) n may be similar to d, i.e. that all the balls in the bag are white.

Laplace assumed that these probabilities have equal chances, thus the value is 1/3 in each; in consequence, we get ten probable ways to draw the three balls.

Now, if we draw three white balls, this means that one of those forms is fulfilled, and since ten of those fifteen forms favour the bag d, the chance is that the bag necessarily contains all white balls: 10/15 = 2/3, and the chance that the next ball to be drawn is white is 12/15. And what applies to the ball n applies to all processes of induction. Therefore, Laplace explained inductive inference on the basis of probability theory and determined the value of the probability of generalisation on inductive basis, i.e.(m+1)/(n+1)and the chance that the next instance have the same quality as (m+1)/(m+2)???.

In his *Positivistic Logic*, Dr. Zaki Naguib regarded Laplace's second equation as a ground to determine the chances that an event be repeated.

Yet, he does not explain the equation in mathematical form, but founds it on an assumption not explicitly proved. "Suppose that a given event never occurred in the past, and that the chance of its occurrence is equal to its non-occurrence, then the value is 1/2. Suppose that it happened once, then the chance that it will happen again is (1+1)/ (1+2) = 2/3.

Then the equal probabilities become three, one of which is positive. The second is also positive and the third is negative. That is, we have two chances that the event will occur and one chance that it will not. In general, if an event occurred m times, this, gives as m chances of its occurrence, and adds two chances, one of which might occur and the other might not"[13]

This quotation clearly assumes that the occurrence of an event more than once favours its occurrence once more, in accordance with Laplace's equation. But this does not justify its frequency. On the other hand, in our exposition of Laplace's theory, we have found an interpretation of the deductive phase of induction different from ours. What is important in Laplace's interpretation is that it dispenses with any axioms save those of probability theory itself. Further, Laplace's interpretation dispenses even with the assumption of causal principle.

Difficulties of Laplace's theory

Laplace's theory involves some difficulties. First, what justification is there for supposing causal chances that the bag n is similar to the bags a, c and d? Or why do we assume that bag n, containing five balls, has three white ones only, and that this is equal to the supposition that it has four or five white ones? The second difficulty is that what justification Laplace has for increasing the probability that all the balls in n are white, because this justification depends on finding out an hypothetical indefinite knowledge which explains increasing chances that all are white. But Laplace's theory has no conception of such indefinite knowledge. The third difficulty is that how do we explain the generalisation of inductive inference on the basis of Laplace's theory?

Let us discuss the second difficulty first. According to our own solution to this difficulty, we suggested there being two sorts of indefinite knowledge, the first sort is the knowledge that n's five balls include three or four or five white balls.

The other sort is the knowledge that drawing three balls from the bag n is taken one of the ten possible forms of drawing three out of five. Again, the first indefinite knowledge includes three members, while the second includes ten members. When the two sorts are multiplied we get thirty forms, ten of which represent the way of drawing three out of five supposing that the bag n is similar to the bag a, the second ten represent the forms supposing that n is similar to c, the other ten represent the forms supposing the similarity between n and d. When we draw three white balls then nine out of ten forms disappear in the first case, or six out of ten disappear in the second case, or none disappears in the third case. Therefore, by means of multiplication we get fifteen forms out of thirty, ten of which favour the similarity between n and d, then the value becomes 10/15 or 2/3 or (m+1)/(n+1).

Such deductive structure depend on the assumption of the two sorts of indefinite knowledge, but it has something wrong, because having drawn three balls from the bag n, we cannot have the second sort of indefinite knowledge, that is, knowledge of one of the ten forms for drawing three balls out of five. In fact, we get a definite knowledge of only one of these forms. This shows that we do not get fifteen probable forms, after drawing three white balls, as Laplace supposed.

Such is the main difference between supposing the determinate choice of a bag n and random choice of the bag a or c or d. In the latter case, we already know that a includes three white balls only, that c includes four white balls only, and that the balls in d are all white. If we randomly choose any of those three bags and draw three white balls, then, the supposing that the bag in question is d, we get fifteen probabilities, and the value would be 10/15.

What has passed does not apply to the choice of n which includes five balls. Here, we get not fifteen forms, but one form. Thus, in the case of choosing the bag n we do not have any indefinite knowledge, on the ground of which we could explain the increasing chances that all the balls are white, and this refutes Laplace's equation for determining the *a posteriori* probability for generalisation.

Further, on reflection, we may discover in the case of the bag n a hypothetical indefinite knowledge, but that does not satisfy Laplace's purpose. When we draw three balls from the bag n and are seen to be white, we cannot obtain the indefinite knowledge which informs us of the increasing value for generalisation. But we can discover a hypothetical knowledge expressed in the following way. If the bag n includes at least one black ball it is either the one drawn first or the second one drawn or the third or the fourth or that it is not yet drawn. Such statement involves five probable hypothetical statements, in three of which the consequent cannot be factually given, since the balls drawn are not black.

Thus we get three probable statements that assert the absence of there being any black ball, and this means that the probability that all n's balls are white is 3/5. And this value is different from that which is determined by Laplace for *a posteriori* probability i.e. 4/6 , for 3/5 is smaller than 4/6. Thus, we have suggested that such hypothetical knowledge does not fulfil Laplace's end because it does not justify the value assumed by him.

If we acknowledge hypothetical knowledge as a basis of calculating probability, we fulfil Laplace's end , such that we may give the deductive step of inductive inference a mathematical interpretation, without recourse to any postulates of causality. The new equation would be that the value of *a posteriori* probability of generalization is m/n instead of (m+1)/(n+1) given by Laplace (m denotes the number of individuals examined, n the whole number of individuals concerned); But such hypothetical knowledge cannot be a ground of increasing probability, because the consequent is factually undetermined. [Here???] we conclude our discussion of the second difficulty facing Laplace by discovering the main mistake underlying his theory.

We now turn to the first difficulty facing Laplace, that is, what justification for assuming the equality among the three probabilities; that the bag n has three white balls among the five, or that it has four or five white ones. We have already remarked the difference between the supposition of the bag n and that of the three bags a, c and d any one of which we randomly take. In the latter case, the three probabilities are equal, while in the former we do not have three bags but only one in which we do not know the number of white balls. Now, if we have no previous knowledge how many white balls and black ones included in the bag n, then the chance that any ball is white is 1/2; the chance that any one is black is also 1/2. Thus, the value probability that n is similar to d is 1/2 x 1/2 + 1/4. n is similar to c is 1/2 x 1/2 + 1/2 x 1/2 = 1/2.

Now, there is no justification for the equality of the three probabilities, hence we do not obtain fifteen equi-probable value as Laplace suggested.

The third difficulty facing Laplace is expressed thus : first his equation cannot determine the value of *a posteriori* probability of generalisation if n denotes an infinite class, because the denominator of the fraction (m+1)/(n+1) infinite, and it is impossible to determine the ratio of finite number to infinite number. Secondly, if n denotes a finite class but has a great number of members, we cannot obtain the probability of generalisation to a higher degree, because the ratio of the members under examination to the total number would be very low.

But our interpretation of probability, hitherto given, supplies a definite value for the probability of generalisation after a small number of successful experiments. For the value of *a posteriori* probability always expresses a certain ratio to the total possible forms for the occurrence of an event or its absence, and such total is always theoretically and factually definite in quantity.

Keynes and Induction

Keynes tried his best to establish induction on purely mathematical lines, by deducing the value of *a posteriori* probability of the generalisation from the laws of probability calculus, as Laplace already did.

Keynes supposes that inductive generalisation has a definite value before inductive process. Let p be the value of such *a priori* probability, and obtaining favourable instances of the generalisation, then the probability of the generalisation after the first instance: p + the first instance; let the sum be p_1 the value, after getting the second favourable instance, becomes: p+ the first two instances, the sum be denoted by p_2. After n of instances, the probability of the generalisation becomes : p+n instances, i.e., P_n.

Suppose we want to know whether P_n continually moves towards 1 (certainty number) as n increases, then it is possible to know that by determining the value of the probability of n instances, supposing that the generalisation is false; let this value be K_n. When K_n moves to zero while n increases, P_n moves to 1 with the increase of n. The value of K_n can be determined through multiplying the value of the probability of the first instance occurring supposing the generalisation is false, in the value of the probability of the second instance occurring.

Suppose n contains four instances for example, and we denote the value of the probabilities by (K_1), (K_2), (K_3), (K_4), we can then say that $K_n = K_1 \times K_2 \times K_3 \times K_4$.

Difficulties of Keynes' Interpretation

Keynes' task is to get a definite value of *a priori* probability of the generalisation, and continually moving it towards certainty, while the instances increase, with the some degree of moving towards zero. Suppose we have a generalised statement e.g., all metals extend value of the probability that every metal extends by heat, before making any inductive process, then through induction we find out the truth of the generalisation. Such process enables us of getting near to certainty. Keynes has two points, first, the determination of the value of *a priori* probability of the generalisation is a necessary condition of explaining inductive inference and its role in reaching the generalisation to a greater degree. Secondly, these are two probabilities (P_n) and (K_n), and the more K_n moves towards zero the more P_n approaches the number 1.

Take the first point first. In the light of our own interpretation of the deductive phase of induction, we may know that the necessary condition of applying the general way determining such phase is to give the inductive conclusion *a priori* probability assumed by the first kind of indefinite knowledge, such that its value does not exceed the value of the probability of negating such conclusion assumed by the second kind of indefinite knowledge. Fulfilling such condition, inductive inference is workable in its deductive phase. However, what is the degree of *a priori* probability of inductive conclusion? If such inclusion expresses causal relation between two terms, the degree of its probability is determined by the number of things thought to be causes or not. If the conclusion expresses causal law, that is, a uniform conjunction between a and b by chance, then what determines the value of its *a priori* probability is the indefinite knowledge consisting of the set the probabilities of a's and b's.

Two attempts have been given by Russell. Russell suggested first a position confirming Keynes in determining the value of *a priori* probability of the generalisation. The generalisation is regarded as merely uniform conjunction. Let us suppose that the number of things in the universe is finite, say M. Let B be a class of n things, and let a random selection of m things Then the number of possible a's is N!/ [m! (N-m)!] and the number of these that are contained in B is n!/[m! (n-m)!]

Therefore the chance of "all a's are B's" is [n! (N-m)!] /[m! (N-m)!] which is finite. That is to say, every generalisation as to which we have no evidence has a finite chance of being true. But this attempt is futile for, first, knowing the number of things in the world[/universe] is practically impossible even if we accepted the finitude of the world; and secondly, the number of things in the world is immensely vast and it is clear that the greater (n) is, the less is its *a priori* probability.[14]

Russell offered another attempt to regard the generalisation as merely uniform conjunction. He suggested that when we regard metals as extended by heat, we find that the *a priori* probability is 1/2, because we have two equal probabilities (that they extend and that they do not), before any inductive process. He suggested secondly that we deduce the value of the probability of "the extension of metals by heat " from the value of the *a priori* probability of any piece of metal. When the probability in the latter case is 1/2, then probability of n extended metals $1/(n_2)$ and this is clearly definite value. This way of determining the value of the *a priori* probability of the generalisation is adopted by us in a previous chapter.

We come now to the second difficulty facing Keynes' theory. Keynes supposed an [xen] any case [of] two probabilities (P_n) and (K_n), that the former gets near the number 1 when the latter moves to zero. The result is that if the occurrence of n (the number of instances favouring the generalisation) becomes less, supposing the generalisation to be false, then the chances increase that n occurs. In fact, this is a deduction of the value of *a posteriori* probability of the generalisation from hypothetical indefinite knowledge. Suppose we have the generalisation that all a is b, suppose again that a has six members, and that by experiment we found that the first four members of a are b; let these be n, and the probability of the generalisation be P_n, and that the probability that four members of a are b, supposing the generalisation false be K_n. Then we can say that knowledge, such knowledge involves that the value of K_n, after four experiments, is 2/5. If n increases in number, K_n decreases to 1/6.

But we have already shown that such sort of indefinite knowledge cannot be a ground of determining probability, because it does not include a consequent in fact. Therefore the value of K_n cannot be determined by such hypothetical indefinite knowledge.

Causal Relations

From what has gone before concerning the deductive phase of induction, we have maintained that the necessary condition of this deductive phase is there not being any *a priori* justification for refuting the rationalistic theory of causality. We are now considering such condition, by discussing another justification for refuting casualty. We may classify the justification of this refutation into four: logical, philosophical, scientific and practical justification.

Logical Justification

The logical justification for refuting the principle of causality rests partly on certain claims, provided by logical positivism. This has maintained that the meaning of any proposition is the way of its verification. A proposition is meaningful if we can affirm it or deny it within experience, otherwise it is meaningless. Now, the proposition all a are followed by b, is meaningful because it is possible to find its truth or falsity through observation. But the proposition a is necessarily connected with b' is different, because necessity adds nothing to mere conjunction through experience, thus experience does not enable us of knowing the truth or falsity of that proposition so positivism asserts that propositions of this kind have no meaning.

We shall later argue that Logical Positivism is mistaken in their conception of meaningful propositions.

Philosophical justification

Empiricism, as an epistemological doctrine, claims that sensible experience is the main and only source of human knowledge, and denies that knowledge has any other source. Empiricism, thus understood, has certain views which may be regarded as a philosophical justification for refuting causality as necessary connection. However, empiricists differ from logical positivists: the latter maintain, as has just been said, that a proposition which we are unable to confirm or refute by experience is meaningless. Whereas classical empiricism admits that such proposition is logically meaningful because the meaning of it and its truth are not identified. Empiricism is satisfied to say that we cannot accept as true those propositions which cannot empirically be verified.

Now, empiricism maintains that causality, as involving necessity, cannot be known to be true through experience, because experience shows us the cause and the effect but not the necessity involved in their connection. That is the point which has been made by Hume. He explained causality as merely concomitance or uniform succession between certain two events. Such empiricistic view has dominated [cent???] thought concerning causality. Instead of regarding it as necessary relation, we consider it as expressing uniform succession among phenomena.

In fact, we cannot[??? can ???] refuse the empiricist view of causality as making experience the criterion of causal relation. But[???] empiricism does not emphatically deny the necessity involved in causal relation rationalistically considered. Empiricism rather implies that such necessity can neither be proved not denied through experience, thus a proposition about necessary connection between events is logically probable. And it is such probability that is needed for induction in order to explain its deductive phase, inductive inference starts from the probability of the relation of necessity between a and b; therefore induction is supposed both by rationalism and empiricism, for the ground of its generalisations.

Scientific Justification

Some scientists have claimed that the principle of causality, involving determinism and necessity, does not apply to the atomic world. But we cannot reject causal principle when we are unable to find a causal interpretation of the behaviour of the atoms. We can only say that our actual experiments do not show yet a definite cause of certain phenomena. At best, this may give rise to a doubt in causal interpretation; and this doubt involves the probability of the truth of the principle, and this is all that is needed for inductive inference as postulate. Further, even if physics has come to the absence of any cause of the behaviour of atoms, it is still possible that causality may probably apply to macrophysical bodies.

Tactical Justification

There is one argument left to justify us in moving from causality (involving determinism and necessity) to causality in the sense of mere conjunction among phenomena. This justification Lord Russell clarifies as follows.

Suppose we have a common sense generalisation that A causes B- e.g. that acorns cause oaks. If there is any finite interval of time between A and B, something may happen during this time to prevent B-for example, pigs may eat the acorns. We cannot take account of all the infinite complexity of the world, and we cannot tell, except through previous causal knowledge, which among possible circumstances would prevent B. Our law therefore becomes: "A will cause B if nothing happens to prevent B". Or more simply: "A will cause B unless it does not". This is a pure sort of law and not very useful as a basis for scientific knowledge.[15]

Now, it is reasonable to offer statistical uniformities instead of rational causality. Instead of saying "A causes B unless it does not", we can say: A is succeeded by B once or fifty times in hundred times. Thus we reach a useful law. Nevertheless, all this does not prevent us from talking in causal terms, on the basis of ignorance. If we are able to know all the things that may prevent A to cause B, we could have formulated causal principle in a more precise hypothetical statement. But that is beyond our reach. For these reasons, we may inquire statistically into the chances according to which A causes B, and then say for instance that A is succeeded by B in twenty cases out of hundred if that has occurred to us in an experiment. But what we are now doing is a generalisation, which itself needs the assumption of causality. Otherwise we fall back in absolute chance, and this cannot be a basis of any sort of generalisation.

We may now conclude that statistical laws are not inconsistent with the assumption of causality because any statistical law expresses a certain ratio of frequency and generalises it, but such generalisation presupposes the *a priori* assumption of causality even in probability terms.

Another Form of Deductive Phase

We have hitherto been considering the deductive phase on the ground that there is an event A inquiring into its cause B. Now, we want to ask first about the very being of A. The latter takes the following forms.

Inductive inference may determine the value of the probability of the existence of A on the basis of an indefinite knowledge which increases the value of the probability of the existence of C. Suppose we say that B has two causes, one of which is A, the other is C. Suppose again that A is a given event whereas C denotes a complex of three determined events d, e and f. When B occurs once, we have an indefinite knowledge that A and C have occurred. Thus we may determine the *a priori* probability of the existence of A with 1/2.

But suppose the probability of the occurrence of A equals that of any of d, e or f, then we get a different indefinite knowledge, which includes the probabilities of the last three events. Such knowledge involves eight probabilities, one of which is the occurrence of all the three events, while the other seven involves the absence of at least one of these events. These seven probabilities implies the occurrence of A because they presuppose the absence of C (d + e + f), and since B has occurred then A occurred. Therefore the value of the occurrence of A is 7.5/8 = 15/16. Here we notice the difference between the two indefinite kinds of knowledge in that each determines the value of the probability of the occurrence of A. Thus we obtain the true value if we apply dominance axiom. Now, in the present case we do not need such application.

We need applying multiplication axiom. Hence we get 16 forms, seven of which are impossible (where assume the occurrence of C without d, e and f); then remain nine forms, eight of which favour A, two favour C (one of these two is common with the eight forms-group) Therefore the value of A is 8/9. We may conclude that the ground of the probability is an indefinite knowledge which increases the probability of C, with the help of multiplication axiom.

Requirements of the deductive phase

It is clear from what has gone before that inductive inference involves generalisation, that is, all A are succeeded by B by virtue of strengthening the probability of causality. Such probability is a result of the probability that there is no other cause of B than A in the first experiment, plus the non-occurrence of a cause of B other than A in the second experiment plus [...] until we reach the final experiment. Each probability shows that A causes B, thus the proposition gets higher probability. But the group of those probability values of the proposition 'A causes B' depends on the condition that a causes b implying the causality of all a's. Such dependence has its justification, because we have already shown that there is a necessary relation between two terms.

Consequently, for induction, in deductive phase, to be performed, it is necessary that several experiments involve many a's between which there is a unity, not just mere grouping. If so, there must be another condition of the generalisation, namely, that there should not be any essential difference between the particular instances of causal relation. For example, if you choose randomly an individual person from every country in the world, and when you notice that some of these persons are white, because this group is arbitrarily chosen and does not have an essential unity. By contrast, if [y[011]???] choose a normal sample of negroes and notice that some of them are black, you can inductively generalise that all negroes are black, because all have a common property.

Successful induction depends on considering natural unities or common characteristics. But this shows that induction involves causality, on rationalistic lines, namely, there being a necessary connection between its terms.

Induction and formal logic

We may now consider the view that evidence for inductive normalisation does not make induction logically valid inference. This view claims that inductive conclusions are not logically necessary. Here we refer to some examples of invalid inductions given by Lord Russell. He classified these examples into two classes, namely, those included in arithmetic and those in physics. As concerns the former, it is easy to produce premises that give true conclusions and other giving false ones. Given the numbers 5, 15, 35, 45, 65, 95, we notice that each number begins with 5 and is divisible by 5. And this suggests that each number beginning with 5 is divided by 5, and that is true.

But if you take some numbers beginning with 7, such as 7, 17, 37,47, 67, 97, they falsely suggest that they are divided by 7 analogically. Further, when we say 'there is no number less than n which can be divided by n', we can enlarge n as we like, thus, we may give the false generalisation that no number is divided by n. Likewise, we can get false inductions in physics when we generalise from a small number of instances.[16]

But it can easily be shown that false generalisations arise from a failure of fulfilling the conditions of induction. In arithmetic, when we make n as large as we can as in the previous example, and find that any number smaller than n is not divided by n, we cannot generalise and apply this property to all numbers, because all common, namely, being smaller than n. If we neglect this condition inductive generalisation, we get false generalisations. Suppose we take a series of numbers beginning with 5 such as 15, 35, 45, 65, 95. Now, what distinguishes this series from any other series of numbers beginning with 5 does not matter, because if we have large numbers beginning with 5, they are still divided by 5, however large they are. Here we may distinguish between induction in arithmetic and mathematical induction which gives us general laws for all integers. In the latter, we have the following steps: applying the law to the smallest number, then applying it to the number (n), finally applying it to the number (n+ 1). This sort of induction is always valid and give us general mathematical laws, and this does not concern us in induction considered in this book.

Let us look at the supposedly false generalisations in physics. Suppose we say "no man whom I now know died", and then say "all living men are immortal". That is a false generalisation because immortality is understood to mean continuous living inaccessible to human limits, and thus it is not applied to men; therefore we cannot infer that men are immortal.

Further, Russell and other logicians have remarked that induction is not valid unless it satisfies certain conditions which give rise to successful generalisations.

Thus, those logicians classify induction into particular and general. Suppose we have before us two classes of things (A) and (B), that we want to know whether a member of A is also a member of B; if by observation we find that all a's are b's then particular induction concludes that another member of A, yet unobserved, is a member of B. General induction shows that all A's are B's. Russell remarks that in particular induction it is necessary to find an instance that verifies our previous observations, that in general induction we should find that all A's are B's, not merely that some A's are B's.[17]

Notes:

[13]Zaki Naguib, Positivistic Logic, PP. 524 -25, Cairo, 1960

[14]B. Russell, Human Knowledge, pp. 426 -7, London, 1948

[15]Russell B., Human Knowledge, pp. 474 -5

[16]Russell B. Human Knowledge, pp. 420-22.

[17]Ibid, pp. 422-3.

Chapter 5. Induction and Certainty

Subjective Role in Certainty

We have hitherto discussed the first phase of inductive inference, that is the deductive phase provided that we accept the axioms presupposed by the theory of probability. But there is a difference between deduction in the inductive process and deduction in purely axiomatic systems, such as we find in geometry. Such difference is clearly shown in that purely deductive systems prove the objectivity of mathematical truths, whereas induction in its deductive phase gives us higher degree of credibility.

This credibility is expressed by greater probability value arising from collecting a greater number of cases concerning the principle of causality. Now, the deductive conclusion in induction shows a degree of credibility of the statement, "A causes B", and not of the principle of causality itself. Such credibility would approximate, but does not reach, certainty. Thus, the deductive phase of induction does not give us certainty in causality or induction generalisation but gives us a greater value of credibility in causality and generalisation [of A causing B]. Now, we want to ask whether such value may reach certainty in a later step of inductive process.

Kinds of certainty

To answer the previous question, we should define the meaning of certainty. There are three kinds of certainty: logical, subjective and objective. We shall explain each in turn. First, by logical certainty, we mean the sense used in Aristotelian logic, the denial of which is self-contradictory. Logical certainty consists of two sorts of knowledge, one of which implies the other. When we say that x is a man' and that 'x is a great man', we say that the latter implies the former. Logical certainty does not only apply to statements but also to the terms of a categorical statement; for instance, straight line is the shortest distance between two points: the relation between the two terms is logical, and it is logically impossible to deny the predicate of the subject. Again, mathematical certainty is a sort of logical certainty because the former involves that a statement implies another.

Subjective certainty is another sort of certainty, which means knowing a statement to be true such that no doubt in it arises. You may deny such certainty without contradiction. I may see handwriting and recognise that it belongs to some friend, but it may really belong to another.

We now come to the third sort of certainty: objective certainty. We may first distinguish the statement said to be certain and the degree of credibility towards it. Suppose I knew that a friend of mine is dead, then the statement 'x died' is certain, but I have also a strong belief of his death; then credibility has degrees ranging from the slightest probability to certainty. In consequence, as to human knowledge, truth and falsity of a statement correspond to reality. But as to our credibility the statement may be true, nevertheless we do not feel certain about it. Suppose someone threw a piece of coin and believed it would rest on its head owing to his desire to be so, and this comes true, then both the statement and his belief are true; yet he would [could] be mistaken in his belief since he foresaw it *a priori*. This shows the distinction between subjective and objective certainty, the former is acquirement of the highest degree of credibility even if there be n[o] objective ground. The latter is the utmost credibility on object grounds. Thus, we may be in a situation in which there is subjective certainty without objective certainty and vice versa. For objective certainty is independent of subjective states of the mind.

Likewise, we may distinguish subjective from objective probability. The latter expresses a definite degree of probability in correspondence with factual data, whereas subjective probability expresses the degree of credibility owned by some individual whether it is consistent with facts or not. We want now to know the objective ground of certainty. A mathematical or logical statement is certain because it is deduced from prior statements; it is the same with objective certainty which is deduced from prior objective certainties. Further, as in formal deduction we start from unproved axioms, likewise in objective certainty, we assume basic or primitive or immediate beliefs. Thus, the objective basis of any degree of credibility presupposes an axiom, namely, that there are degrees of credibility immediately known to be objective. Thus we have two sorts of deduction, deduction of statements and deduction of the degree of credibility. The statement the internal angles of a triangle are equal to two right ones is a deduction of the former, while when I throw a piece of coin and I say it would rest on its head or tail I talk of the latter. Now we want to ask whether the value of probability involved in inductive [inclusion???conclusion] could be transformed into certainty, and to this we shall turn.

Certainty which is required for induction cannot be logical certainty because if we say that a causes b it is not logically impossible to suppose that a does not, or that b can have another cause than a; that a causes b cannot be deduced from our observations. Again, subjective certainty is not required here because the majority of men take it to be beyond doubt. It is objective certainty which is required for induction? Now, is there any justification for saying that inductive inference may be objectively certain?

Objective certainty require[s] an axiom

We have hitherto remarked that valid and objective degree of credibility can be deduced from other objective degrees of credibility but such objective degree can be immediately given; we have remarked also that objective credibility requires an axiom, namely, that there are degrees of objective credibility immediately given. These degrees being given are not deduced from other prior degrees of credibility but this does not mean that degrees of credibility not deduced from others are primitive and immediate, because we may find certain subjective degrees of credibility, as opposed to objective credibility that cannot be deduced from prior assumptions.

What distinguishes those objective degrees of credibility immediately given is that they are consistent with each other, such that the least inconsistency among them shows that they include something subjective. Thus one way of finding out that a certain degree of credibility is not immediate axiom is to show that it is contradictory with some objective degrees which have general acceptance. For example, suppose someone believes that the book I want to buy is not the one missing in my own study, then I can show him that the degree of his belief is subjective not objective, that his degree is not primitive and immediate, and that it is contradictory with some other beliefs, such that he strongly believes that there is a book missing and I want to buy another copy but what to buy is not the missing one. Now, his belief that any book I want to buy is not the missing one does contradict with his belief that there is one book missing in my study. Therefore his degree of credibility is subjective not objective.

If we are concerned with induction in this light, we find that the degree of credibility required for the inductive conclusion is objective, being derived from other objective degrees provided that such degree is always less than certainty. The inductive conclusion cannot reach certainty because there is always a value of probability which makes the difference between the probable and the certain. Now, for induction to reach the highest degree of objectivity, we must assume that there is a degree of credibility, immediately given, and such assumption is needed for any deductive process, which depends on an assumption, namely, that there are certain objective degrees of credibility, not deduced from prior degrees. Although we accept this assumption, we cannot prove it, nor can we prove any similar assumption. We cannot even prove that highest degree of believing the law of non-contradiction is one that is immediately given. And if we approve this assumption, we have three points to consider.

First, we must give a precise formula of the assumption, for induction to reach the highest degree of objectivity, that is, certainty. Secondly, we must specify the necessary conditions required for the assumption to be valid and avoid falsity. Finally, we must be sure that these conditions are to be fulfilled in the objects and concepts hitherto studied in the first deductive phase of induction, and thus, it is possible for induction to proceed to its second phase, i.e., to reach certainty.

The formulation of the postulate

The postulate presupposed by induction in its second phase is concerned not with objective reality, but with human knowledge itself, and can be stated as follows.

When a great number of probable values are reached in a specific inquiry and a larger value is obtained, this is transformed into certainty. For human knowledge is so constituted that it does not satisfy with small probable values. That is, the postulate assumes that neglecting smaller values in favour of larger ones which would come to certainty is a natural inclination of human knowledge.

When we move from higher probabilities to certainty, we do not rely on psychological factors as optimism or pessimism. The probability of the death of a person about to have a surgical operation could move to certainty as a result of pessimism on his part, but such certainty comes to an end if the person in question gets rid of his pessimistic state. Whereas the certainty involved in our postulate is one that cannot be an illusion.

We have already claimed that probability values are always connected with an indefinite knowledge, and that any probability value is one of a member of an indefinite knowledge; thus when the impact of probability values in some inquiry comes to the degree of certainty we face an indefinite knowledge which absorbs most of those values. Now, we may ask about the limit of the greater value which could become certainty and the limit of smaller values which would be ignored. People have transformed probability to certainty. Some think that they get certainty when probability value in a certain inquiry reaches a certain degree, whereas others do not think such degree satisfactory. But it is not necessary for the postulate required for induction to determine the degree which is a sign of certainty, but it is sufficient for the postulate to state the principle that the increasing number of probability values in a certain inquiry indicate the transformation of probability to certainty, and that the required degree is involved in successful inductions.

Conditions of the Postulate

The postulate under discussion stated that when probability values reach a certain degree and involve a greater number of cases, these values absorb smaller values and transform probability to certainty. But there is a [principal???principle] condition for the postulate to work, namely, that the passing away of lesser values should not to be with it the passing away of higher values. Take for example the case, of the missing book in a whole library containing 100,000 books. Here we have an indefinite knowledge that there is a book missing in this whole library, this knowledge involving bundled thousand probability values, and that each value is equal to 1/100,000, that is, the probability that anyone of these books is missing. Now, if we take anyone of these books we find that the values of presence of book equal all the values connected with all other books present in the library.

It follows that such book will be the beginning of our inquiry into the probability values of the present books, save one. But such greater value does not transform probability to certainty because the impact of greater values is nothing but an expression of the greater part of our knowledge of a missing book, and any book that is supposed present has value equal to the value of the probability of the missing one. In this case, the lesser value did not pass away in favour of the greater values of the present books; if it does, either this leads to the passing away of the smaller probability value opposite to all other values, or opposite to some values only.

Thus we maintain that it is impossible for the postulate to be acceptable. In consequence, the higher values cannot be transformed to certainty, since we presuppose one indefinite knowledge otherwise we always face probability values equal to the number of the terms of such knowledge. Now, if we want to make the postulate acceptable, we stipulate two sorts of indefinite knowledge. This takes two forms, which we shall presently state in detail.

The first form of the postulate

We suppose that we have two sorts of indefinite knowledge and that probability values focus in one direction. We may state this [section negatively by negating a definite in term in the indefinite knowledge 1???], and positively by affirming another term of the same sort of knowledge. But those values associated in one direction belong to the indefinite knowledge 2. Thus, the association and the direction of it do not belong to one sort of knowledge but to two. Let us apply such form of the postulate to the proposition "A causes B" in view of two sorts of indefinite knowledge. First, the indefinite knowledge which determines the *a priori* probability that A causes B. If we assume that we have already known that B has a cause, that this is either A or C, then such knowledge includes two terms, let us call it knowledge 1. Secondly, the indefinite knowledge just stated may be taken as a ground for establishing the probability of causality thus such knowledge involves all successful cases where in C is probably a cause, let this be called knowledge 2.

If we get ten successful experiments, then we should have 1024 cases, being the terms of knowledge 2; one case of these is indifferent to the two terms of knowledge 1, the remaining cases favour one of the two terms in knowledge 1, that A causes B.

This means that knowledge contains 1024 probability values, that 1023 1/2 values constitute a positive grouping in a certain direction, namely, that A causes B, one of the two terms knowledge. This grouping gives the causal relation concerned higher probability. Now, we may validly apply our postulate to such a case. We postulate that such grouping of cases gives us certainty as to the causality of A to C, and the passing away of the contrary value. Such application involves no contradiction because this grouping expresses the greater part of knowledge 2. Now, we get the first form of applying our postulate: when a number of probability values of an indefinite knowledge increases outside limits, and leads to the passing away of the contrary value.

Such application involves no contradiction because this grouping expresses the greatest part of knowledge 2. Now, we get the first form of applying our postulate: when a number of probability values of an indefinite knowledge increase outside its limits, and leads to the passing away of one value, then this latter does not belong to the knowledge connected with the great number of values.

But there are two conditions to be satisfied for this application to hold. First, the proposition, expressing the vanishing of a probability value as opposed to the greater number of values, should not be concomitant to one of the terms of knowledge 2 to which belong those values. That is, if 'A causes B' is concomitant to the occurrence C in all successful experiments, and we know that if C occurs in all cases, then A is not the cause of B necessarily. This makes the application of the postulate difficult, because 'A causes B' would become one of the terms of knowledge 2. If the terms included in knowledge 2 lead to the vanishing of the value of causality, then they naturally absorb the probability value of C's occurring in all cases. And then the postulate is faced with the problem of certain values absorbed in favour of other values without justification.

For the application of the postulate to succeed, we must assume that the proposition, expressing the absorption of a certain value, is not concomitant to one of the terms of knowledge 2, as this in fact is the case of refuting 'A causes B'. Such refutation is not connected with the occurrence of C in all experiments. For C may occur in all experiments and yet B is caused by A.

The second condition is this, that the grouping of probability values must not be arbitrary; by grouping we mean that values are not related immediately to one proposition but some values are related to a proposition while others are related to another, and from these two propositions stands. This third proposition is called arbitrary. For example, suppose we put a very heavy stone on the top of a pillar under which some one is sitting. If the stone is properly put on the top it does not fall to the ground, but if improperly put, then we have an indefinite knowledge that the top has, say, thousand points. Such knowledge includes 999 values involving the fall of the stone and the death of the person underneath, and only one value that it does not. Now in view of the conditions stipulated for the postulate we may know of the great number of other values because all belong to one sort of knowledge. Here comes the arbitrary grouping which is supposed to overcome the difficulty. We may explain the certainty required by grouping the probabilities according to our postulate without the assuming that the one value referred to above will cease to stand.

Instead of assuming the certainty that the stone will fall we suggest another proposition, namely, that it is probable that the person in question dies from a heart attack not owing to the fall of the stone.

Then we find a third proposition, namely, that either the stone falls or the person dies from heart attack. Thus we falsely get the certainty that the person should die. If we do this, we cannot apply the postulate without contradiction, because if we suppose that the probability of the death of person from heart attack is equal to that of the person being kept alive. In such case we have two propositions having the same probability value: first, that either events would occur, the fall of stone or death from heart attack; the second is that either events would occur: the fall of the stone or that the person will not die from heart attack. Theses two propositions are equally probable, and this proves that the greater probability value arising from the grouping of values in the first proposition cannot be transformed to certainty, for if it can, then this would be without justification; and if the values of both propositions become certain, this means that we are certain of the falling of the stone, but we assumed this not to be so.

And if we assume that the value of the probability that the man would die out of heart attack is greater than the value not of his death, some events may possibly occur such that each one has an equal probability of the man's death out of heart attack. This may substantiated as follows:

1) Either the stone falls or the man dies out of heart attack.

2) Either the stone falls or rain falls.

3) Either the stone falls or the temperature increases.

Those propositions are equally probable assuming the equality of the probability of death, rain and temperature. This shows that the probability value of the first proposition cannot be transformed into certainty. And we know that the value of the occurrence of any of the three events is greater than not occurring, we may sometimes get a value higher than the value of occurring, by means of grouping the values of not occurring. All this shows that the application of the first form of the postulate in an arbitrary direction is self-contradictory, for the postulate to be acceptable it must be applied in a definite direction, and by this is meant a proposition that directs the probability values to confirmation such as the proposition that A causes B in the above example.

Objections and Answers

1. Is causality a term in indefinite Knowledge

An objection may be raised as to the application of the first form of postulate to causality, namely, that the example of causal relation, referred to above, does not fulfil the necessary condition of the postulate, that the proposition expressing improbability in favour of grouping a greater number of probability values must not be a term in the indefinite knowledge concerned, because such proposition must refute that A causes B, but this refutation is itself a term of that knowledge thus it cannot be absent in the latter.

This objection depends on the rule of multiplication, represented by the principle of inverse probability, and is irrelevant if we have in mind the rule of dominance which is an application of the third additional postulate, explained in a previous chapter this rule says that knowledge 2 is the sole ground of all the values which the improbability of A causing B, and the refutation of causality is not included in that knowledge.

The answer to this objection, provided we use the rule of multiplication instead of dominance rule, is as follows. The appearance of knowledge 3, resulting from multiplying the members of knowledge 1 in those of knowledge 2 depends on keeping in itself all the members of the other two pieces of knowledge. In such a case multiplication includes a number of probable instances which form the members of knowledge 3, and the negation of causality is substantiated in these members. But the postulate we suggest for induction in its second phase assumes that probability values grouped in knowledge 2 are inconsistent with the probability that A is not cause of B. Such postulate led to the passing away of the improbability of c causality, then knowledge 1 no longer includes both members to be multiplied in the members of knowledge 2. Thus knowledge 3 would not arise.

2. Attempt to deny our knowledge of causality

Another objection may be raised against our certainty about causality arising from the grouping of probabilities, according to our postulate. This objection is meant to argue that the postulate is false.

Let us make the objection clear. When we know that something is the case and we doubt in something else, then we may affirm what we know, whatever we say of what we doubt. For example, if we know that rain in fact falls, and doubt whether there is eclipse, this means that we are sure of the former, and our doubt of the latter does not affect the fact of raining. Our knowledge of the rain fall involves that of two hypothetical statements, namely, knowing that if there is eclipse, the rain falls, and knowing that if there is no eclipse rain also falls. That is, rain falls whether there is eclipse or no, and if we do not know these two statements we cannot know that rain falls.

In this light, if we analyse the conclusions we arrived at in the previous application of the postulate, we find that we have in fact got the knowledge that A causes B, and that we have got the probability of the occurrence of C, because such probability, however lower its value, cannot be ignored being a term of knowledge 2. If those conclusions were true and we are certain that A causes B with a doubt in the occurrence of C, it would be necessary that our; knowledge of causality involves two hypothetical statements, as has been already stated. Knowledge 2 is clearly not existing, that is, we do not know that A causes B provided that C has occurred in all experiments.

Thus, had we in fact observed C in all experiments, we would not have been certain that A causes B. This means that the refutation of causality is probable, provided that C always occurred, and since this hypothesis is probable the refutation of causality is probable. We may answer this objection as follows. Certainty about some fact may arise when we prove that it is the case of when we group probability values according to the postulate of inductive inference. The first sort of certainty affirms the fact, whether the other events occur in fact or not; thus it is impossible to group something certain together with our doubt in it. The second sort of certainty, arising from grouping a great number of probability values is not strict certainty, even if we assume the falsity of one or more values, for such assumption involves the falsity of some of those grouped values.

Now, certainty about causality, being a result of the grouping of a greater number of probability values, cannot be affirmation of causality if we assume that such values were false and that C occurred in all experiments. Hence, any inductive certainty about some fact, resulting from greater probability values, cannot be certain knowledge if it involves doubtful values; thus we cannot prove that causality is inductively not certain, if we suppose that C occurs in all successful experiments.

3. Misapplication of inductive postulate

We may imagine a third objection stating that we sometimes give an application contrary to the postulate itself. This comes out when we get probability values contrary to the phenomena under examination but equal to the favoured values, e.g. the examination whether A causes B or not. In such a case it is impossible that negative values (negation of causality) would be superseded owing to their lesser degree. For then the result that A causes B would have no justification since the values, positive and negative, are equal. If C occurs concomitant to B in all experiments then it is not probable that A causes B, and if C does not occur in all experiments then it is not the cause of B. Thus we reach the absurd conclusion that knowledge 2 has superseded the probability value of one of its terms i.e., the probability that C occurs in all experiments.

We may simplify the previous objection as follows. We assume first that causality is verified inductively; we assume secondly one of two alternatives: either C did not happen even once, or that A is not cause of B. In both assumptions, we have values in the one equal to those in the other, and both belong to knowledge 2. Now if we suppose that this knowledge involves certainty about the first assumption without the second, then the supposition is a probability without justification. And if we suppose that knowledge 2 involves certainty about both assumptions, this means that we are certain that C does not occur in all experiments, thus knowledge 2 has superseded one of its terms.

It may seem that we answer the objection is terms of the second condition stipulated for the application of the postulate, namely, that the objective of the inquiry should be real not arbitrary. We may say that we encounter the refutation of causality with an arbitrary objective which is a complex event, that we encounter the affirmation of causality with an arbitrary objective which is choosing an alternative, as in the example of a stone falling from the top of the pillar on a person underneath.

But the postulate depends on a real objective, that is, affirmation or negation of causality. But when we have done this, we have proved that such application is self contradictory.

The present objection tries to include the real aim of inquiry as arbitrary, and show that applying the postulate to both real and arbitrary inquiries leads to contradiction. Now, to get rid of these contradictions, the postulate must be concerned with the real, not the arbitrary, inquiry. This we do in what follows.

Indefinite knowledge 2, with all its probability values, positive and negative, is directed towards our certainty in causality. If this knowledge fulfils such certainty, then causality is affirmed, that is, there would be no probability that C occurs with B in all experiments, together with the non-occurrence of A. It has been shown that part of this complex event does not happen; in consequence, the probability of Cs occurrence in all experiments is associated with the probability of the concomitance of A and B. Thus, the arbitrary inquiry would be part of knowledge 2; and in this case the postulate does not apply, so long as we stipulated that the postulate cannot apply to those cases in which indefinite knowledge supersedes some values, in favour of others.

We now realise that applying the postulate to the real investigation expels the arbitrary one out of its domain because the former becomes one of the terms of indefinite knowledge itself. But if we suppose the application of the postulate to the arbitrary investigation, this would not extract the real investigation out of its own domain.

For the application of arbitrary investigation leads to a knowledge that the complex event does not occur-the event consisting of the occurrence of C in all experiments and the concomitance of A and B. That A causes B, and that C uniformly occurs with B cannot happen at the same time. And that is the true course of application. For the certainty that A's causing B together with C's occurring do not happen and this does not make us believe that the denial of causality is connected with the uniform occurrence of C, otherwise we would believe the hypothetical statement: 'if C uniformly occurs then A is not cause of B' but we are not certain of the truth of this statement.

The outcome of all this is that the postulate can explain the real investigation of causality, and cannot explain arbitrary investigations.

4. Indefinite Probability

We may state the last objection to applying our postulate. The postulate supposes that the grouping of many probability values contrary to the investigation concerned rules out the latter's value. Suppose also the probability that A is not cause of B to be ruled out owing to the lesser values favouring it. But if we add all causal relations which we inductively arrived at, and observe the probability that at least one of these relations, may not occur in fact, then we find this latter case more probable that the denial of all relations.

For the assumption that at least one causal relation cannot occur is not ruled out by the affirmation of many causal relations, but it can be ruled out by multiplying those values. Thus we find that the probability that at least one causal relation does not occur, this probability remains as probable as indefinite knowledge. In this case it is impossible to apply the postulate to all domains of induction, because if it is applied to all domains it would rule out the probability that a certain value does happen, whereas we assumed that such probability cannot be ruled out according to the postulate. And [11???] the postulate be applied to some, but not all, causal relations, the application is. without justification.

Answer. The probability value here is a consequence of the addition of all values refuting causality, and the postulate is capable of ruling out such values.

The Second Form of the Postulate

In considering the first form of the application of the postulate, we have confronted two sorts of knowledge, the first absorbs the probability value of one of its terms (knowledge 1), the other is the cause of this absorption in virtue of the grouping of a great number of values in a single investigation, and thus we avoid the supposition that a sort of knowledge absorbs or rules out one of its values being equal.

But in the second form of application, we will suppose that the sort of knowledge, which rules out the lesser value in favour of the greater value in one single investigation, is the same knowledge which rules out the probability value of one of its terms. That is, though our knowledge absorbs the value of this, we are not led to wipe out knowledge itself or to rule out some values in favour of others, both being equal, without justification. For, in the present application we assume that the part of knowledge, the value of which is ruled out, is not equal to the standing part, but smaller than other values. Thus, the application of the postulate is not confronted with any difficulty such as we have established something without justification or the vanishing of knowledge itself. It is possible to suppose the postulate to rule out our knowledge of the lesser values without falling into reaching a conclusion without justification because the justification of ruling it out is its being small in content.

Still we have an essential point to explain, namely, how do probability values in a certain sort of knowledge differ from each other though the importance of such knowledge lies in the various values being equal, as we have seen in the theory of probability? Such difference in the values involved in an indefinite knowledge must be understood in virtue of another indefinite knowledge.

Let the former be called indefinite knowledge 1, and the latter knowledge 2. This last knowledge must offer an unequal distribution of the values of knowledge 1; and this is done by one of two ways which we shall illustrate in two examples.

We suppose first the occurrence of events, let them be three events. We notice inductively that the cases of the occurrence of each event are more frequent than not.

Then the probability that the three events would not occur is lesser than other values. For example, suppose we find in the newspaper that the cases of true news is twice the cases of false ones. Suppose we have before us three news, then two of them are supposedly true and the third is false. Then we get two sorts of indefinite knowledge, knowledge 1 and knowledge 2. The former includes eight probabilities concerning those three news and their truth and falsehood, one probability of which would be the falsehood of the three news. Within this sort of knowledge the values of these eight probabilities are equal, namely, the value of each is 1/8. But knowledge 2 would include nine probabilities, three concern the truth and falsehood of each of the three news, on the supposition that the cases of truth are twice those of falsity. That is, knowledge 2 includes the being of a case in each of the nine probabilities, thus we have 27 truth values, one of which involving the falsity of all cases, and the rest involve the truth of at least one value. Thus knowledge 2 changes the probability values of knowledge 1, then making the values unequal.

There is another way of applying the postulate. We may suppose a group of events, the occurrence of which is equal to its non-occurrence, thus we get equal values of all the possible probabilities of the occurrence of non-occurrence. These probability would include the indefinite knowledge. Yet some of these probabilities are correlated with one opposite case, included in knowledge 2, such that the latter case is lesser in value than the other cases. For example, if we throw a piece of coin ten times, we find that it is probable that the coin is on its head or is not. The two probabilities are equal, by multiplying them in each throw we get 1024 probabilities, and these constitute the indefinite knowledge 1. Within this knowledge, the values are equal. For instance, the occurrence of the coin on its head in the first, fourth, ninth and ten times, and on its tail in all times must be equal.

Yet it is known that the occurrence of the second case is strange enough, while the occurrence of the first case is not. This means that there is one factor which makes the second case less in value than the other case. Such factor shows the importance of knowledge 2.

Now, what is this factor? It is this factor which made ancient formal logic believe that it is impossible to regard the uniform occurrence of certain events as chances. Formal logic denies that chances consistently recur in a great number of experiments; for instance we do not expect a piece of coin to rest on its head in thousand throws.

However, formal logic wrongly explains why this is impossible. Aristotelians rejected chances on *a priori* principles, but recurrent chances may well be explained in another way.

If we have before us an experiment, say throwing a piece of coin, in a number of cases, and compare them, we shall find that these cases have more differences among each other according to the circumstances belonging to each than the circumstances they have in common; instances of the latter are the direction of air, the position of the hand, and other circumstances which may interfere and direct the experiment. Now, if we suppose the coin to rest on its head a number of successive times by chance, this means that the items of circumstance which are permanent are the cause of what happens.

Then when we observe the recurrence of some factor we explain this by the permanence of the circumstances belonging of the coin. Such explanation have a very small value, because the changeable circumstances are more numerous, and each of these may be a factor in directing the experiment.

Take another example. Suppose we invite fifty persons for dinner and predict beforehand the colour of costumes they will wear, we will find that the probability of their coming wearing their costumes in one colour very small. For the choice of the costume for each person differ according to their personal circumstances being vastly different. If it happened that they all come with costumes having one common colour, this means that the few circumstances they have in common are the cause of what happened. Thus the probability is very small if they wore costumes having one and the same colour.

The truth of the matter in both examples is there being an indefinite knowledge that the cause of the coin's resting on its head, or choosing yellowish costumes, is either one of the various circumstances involved. And the items of this knowledge (which we call knowledge 2) are more numerous than those of knowledge 1, because the latter derives its items from the various forms of probable situations. But knowledge 2 derives its items from the number of circumstances related to the first situation multiplied with those related to the second situation and so on. Thus we find in knowledge 2 a great number of probability values which oppose the supposition of consistent chances.

In consequence, we see in good light the Aristotelian principle that "chance does not recur uniformly and regularly". This principle is in our view, not *a priori* or logical rules, but a grouping of probability values such that the probability of uniform chance is very low.

Reformulation of Aristotle's principle

We may now reformulate this principle after ruling out its so called *a priori* character in the following way:

(1) We are aware of a great number of varieties between a certain point of time and other points, and between any given state in nature and others;

(2) We are also aware of a small number of resemblances between any two points of time or any two physical states.

(3) Such awareness makes the values of these varieties very great and significant;

(4) If the first point of time, or the first state of a given object, leads to an event which we cannot at the time know its cause, then our expectation of the next point of time or state to bring the same event by chance is much less than having a different event. Let us call the principle reformulated "the rule of irregularity".

We must notice that we presuppose that the interference of changeable factor producing a certain event involves variation and difference from state to another; thus we regard the value of the event occurring regularly equal to the value of having the permanent, not changeable factors which produce the event.

Such presupposition may be confirmed inductively: the difference between two things is connected with the difference between the conclusions. The inductive confirmation of the presupposition is to suppose it certain.

This means that we have applied the first form of the postulate in its second phase, by means of which we arrived at this inductive statement. But in order to explain the difference between the values of the items of knowledge 1, it is not necessary to obtain the inductive confirmation of this statement as certain. It is sufficient to confirm it with higher probability. That is, the supposition of the effect of the variations and changeable factors on the event implies the non-recurrence of this event uniformly in every case.

Now, in disclosing the Aristotelian principle rightly formulated as the grouping of probabilities, we can explain certain vague points in the way of applying such principle. First, the principle of irregularity involves that the uniform recurrence of chance is improbable, provided that the so called regularity is real not artificial. By real regularity, I mean the regularity which shows a common cause, as when the coin rests on its head two successive times; this means that the circumstances common to both cases are the same.

And since the differing circumstances are more numerous than the common ones, the probability of the uniform recurrence of chance occurrences is very low. By artificial regularity, I mean the regularity which does not involve a common cause. Suppose that when we throw a piece of coin, someone expects randomly that it will rest on its head in the first throw, on its tail in the next throw, on its head in the third and on its tail in the fourth [...???] and so on till the tenth throw.

In this case there is an artificial regularity because we do not suppose the cause of the coin's resting on its head in the first throw is the same cause of what happened in the second throw. Thus the probability of the truth of this expectation by mere chance all the time is not lower than any other probability. We notice in fact that the probability of the truth and falsity of the expectation are equal.

Secondly, the rule of irregularity doubles the probability value of chance repetition in case of real regularity, as we have already seen; and so long as regularity between supposed chances are clearer, the rule of irregularity is more successful. Suppose we are told that someone (x) made ten trips in the course of ten months, and in each trip he happened to have a road accident; this would be strange. But if we are told that he made ten trips in one month and in each trip he had an accident, this would be stranger. Again, if x invited ten friends and it happened by chance that all of them were ill then none come. This would be stranger than supposing that he invited ten friends in ten months time but one friend could not come in the first invitation, another friend in the second invitation and so on. In both cases, we have regular chances that the first is the more regular than the second, on the ground that all happened at the same time. This means that when real regularity of chances are more consistent and uniform, then the probability of their occurrence all over is low. For the circumstances which each invited has are naturally different according to the physical, psychological and social factors each of them has.

Likewise, differences in circumstances of the ten persons are much clearer than the agreements in their states. Consequently, it is strange to judge that all are caught with a headache at the same time in spite of the great variation in their circumstances. But it is less strange to judge that a friend was ill in this month and another was ill in the last month and so on.

We may recapitulate. The second form of applying the postulate of induction presupposes two sorts of indefinite knowledge; knowledge 1 includes all probabilities of occurrence and non-occurrence in respect of certain group of events. These probabilities have equal values in that knowledge. Knowledge 2 helps to change those values in two ways. First, knowledge 2 makes the Probability of the non-occurrence of any event in the group less than that in knowledge 1 provided that the probability of the occurrence of each event is more than its non-occurrence. Second, knowledge 2 makes the occurrence of all events in the group the least probable, provided that these events are really regular, then by means of the rule of irregularity, the probability of regular events in each time is very low, and when there is no such equality of values involved in knowledge 1, knowledge 2 can take the least value a centre of probability for the contrary, and this will be ruled out according to the inductive postulate.

Discussion

The first way of applying the postulate does not work and is insufficient. For instance, if we suppose that the motives of saying truth is double those of telling lies, then 2/3 of the news are true and 1/3 are false. Suppose we randomly collected a thousand items of news, which form knowledge 1 having all news true and false so far, the probability of the falsity of all news is equal to any other probability. But if we introduce knowledge 2 we get all possible probabilities involving the motives of truth and falsity. The items of knowledge 2 are more numerous than those of knowledge 1, because any probable event in knowledge 1 corresponds to three probable events in knowledge 2, provided that every piece of news has three motives, and the probability of getting the motive of lie as one of the three motives relative to each piece of our news, includes contrary values larger those included in knowledge 1. Thus, the probability of the falsity of all false news is the lowest probability included in knowledge 1.

All this is acceptable, but it is not regarded as a true application of the inductive postulate and rules out small probability values of the falsity of all news, nor does it give rise to the certainty that some of them at least are true, though the application does not contradict knowledge 1, since the probabilities meant to be ruled out is lower than any other probability. But the application is inconsistent with another sort of knowledge, that is, we usually know there being of thousand false news in the whole body of news we have- this may be expressed in an indefinite knowledge 3. The items of this knowledge include every group consisting of 1000 news the truth or falsity of which we do not know. It involves the equality between any false group and any other.

In consequence, if we randomly choose 1000 piece of news out of the whole news, and by virtue of knowledge 2 including the determination of the values of knowledge 1, made the probability of the falsity of all news in its lowest degree, this does not justify the application of the postulate and reach certainty in justify the application of the postulate and reach certainty in opposition to that probability. For if we apply the postulate to the first thousand news only, this is without justification; and if we apply it to the whole news we have, then we face an indefinite knowledge of 1000 false news. Thus it is impossible to apply the postulate according to the first way of introducing knowledge 2 because this is inconsistent with knowledge 3, and this leads to certainty without justification or ruling out the values of knowledge 3.

If we replace knowledge 3 by a relatively greater degree of probability, we reach the same conclusion. For then we suppose we do not know there being thousand false news, but we have only reasonable probability. Such probability is an indefinite probability. If we apply the postulate to all groups of thousands we rule out this probability though it is of reasonable degree, thus we cannot rule it out according to the postulate.

It may be mentioned in this context that indefinite probability differs from the probability referred to in our fourth objection to the first form of applying the postulate, i.e., the probability that at least one causal relation is not constant. For such probability is a result of grouping the probabilities of non-causality. But here the probability replacing knowledge 3 is not a result of such false probabilities; it is the ratio of falsity of the news to truth. Therefore the first way of introducing knowledge 2 is insufficient for reasonable application of the postulate. But the second way of application is more sufficient.

Objection and Answer

It may be objected to this second way of applying the postulate that introducing the change of values of knowledge 1 and disappearance of the lowest of these values, give rise to the disappearance of the value of one of the items of knowledge 2. For example, if there is no probability that a piece of coin rests on its head in thousand successive throws; this means that knowledge 2 will also lose one of its items which is the supposition of something in common in the course of different throws and this something would be the factor determining the coin's head or tail. Thus the application of the postulate is self-contradictory because knowledge 2 negates one of its probability values which are equal. In consequence, knowledge 2 rules out either one of the equal values or all its values.

In answer to this objection, we argue that the disappearance of the small value in knowledge 1 is made by virtue of the effect of the number of value in this sort of knowledge, not by means of the opposite values in knowledge 2. Now, the values superseding the probability of the coin's resting on its head in thousand successive throws within knowledge 1 would be sufficient and of greater value. But here there is an obstacle, namely, that the value of the coin's rest on its head all the time equals any other probability value within knowledge 1; and there is no justification in this knowledge ruling out such value. Thus the postulate requires knowledge 2 to lessen the value of the coin's appearing on its head all the time, and then overcome the obstacle.

Finally there is a further objection, namely, that it is possible to prove that if there is any value the frivolity of which is presupposed by the inductive postulate, by reason of its unimportance, we can find equal values which are not frivolous though unimportant, and this shows that the unimportance of a certain value does not and itself in value that it is ruled out. The argument adduced by the objection is stated as follows. In any value the ruling out of which is presupposed by the postulate, we can suppose an indefinite knowledge consisting of a great number of items so that the division of certainty number on these items equals the indefinite value supposed to be ruled out. For we know that the value of any of these items cannot be ruled out however we increase or obtain a value which cannot be ruled out.

There is an answer to such argument. When we have two equal probability values it is impossible to rule out one of them and keep the other, for this would be without justification. But if the grouping of data relative to a certain value belongs to an indefinite knowledge while the data relative to another equal value belongs to a different knowledge, then it is possible to rule out one of the values in favour of the other.

After having considered in detail our new interpretation of inductive inference, we come now to study the theory of knowledge and its main topics. We shall take, as our ground, intuitionism in a certain sense to be later specified. We shall start this task with a brief exposition of Aristotelian theory of knowledge.

Principles of demonstration

Formal logic claims that the objects proper of human knowledge are those which involve certainty, and by certainty is meant by Aristotle knowing a statement beyond doubt. Certain statements are of two kinds :

(a) statements which are conclusions of prior certain ones;

(b) basic statements regarded as ground of all certain subsequent statements. Formal logic classifies those certain statements into six classes:

(1) Primitive statements - the truth of which the mind admits immediately such that the apprehension of the terms is sufficient for judging their truth, e.g., contradictories cannot both be true or [xof] that the part is smaller than the whole.

(2) Basic empirical statements - the truth of which we admit by sense-experience; these come to us either by outer sense, e.g. this fire is hot, or by inner sense, e.g., we are aware of our mental states.

(3) Universal empirical statements - the truth of which is admitted by the mind through repetitive sense perception, such as fire is hot, metal extends by heat.

(4) Testimonial statements - the truth of which we believe upon the testimony of others whose utterances we believe true, e.g., such as our belief in the existence of places unobserved by us.

(5) Intuitive statements - the truth of which is believed in virtue of strong evidence that dispels any doubt, such as our belief that the moon derives its light from the sun.

(6) Innate statements - these are similar to primitive statements except that the former needs a medium approved by the mind such that whenever an innate statement is present, the mind understands it by the aid of something also. E.g., 2 is half 4 because 4 is divided into two equal numbers, and this means its half.[18]

Any premise derived from any of these classes of statements is also certain. Those classes are considered the basis of certain knowledge, and the premises derived from them form the body of knowledge.

Any derivation in this structure takes its ground from the correspondence between our belief in originally certain statements and our belief in their derivatives. Such structure of knowledge is called, in Aristotelian terms, 'demonstrative knowledge', and the inference herein used is called 'proof'.

Principles of other forms of inference

Principles of inference in formal logic are not confined to those of demonstration or not confined to the those of demonstration or proof, but there are also other principles inference such as probable commonsensical, acceptable, authoritative, illusive and ambiguous statements. There are classes of statements out of which one can establish uncertain inference. Let us make such classes of statement clear.

(1) Probable statements: those which admit either truth or falsehood, e.g., this person has no job therefore he is wicked.

(2) Commonsensical statements are those which derive their truth merely from familiarity and general acceptance, e.g., justice is good while injustice is bad, doing harm to animals is vicious.

(3) Acceptable statements are those which are admitted as true either among all people, or among a specified group.

(4) Authoritative statements are those admitted by tradition such as those come to us from holy books or sages.

(5) Illusive statements are false ones but which may be object of belief by way of sensual evidence, e.g., every entity is in space.

(6) Ambiguous statements are false ones but may be confused with Certain statements.

Now, all inference depending on certain statements is called demonstration, but when inference depends on commonsensical and acceptable statements it is called dialectic; and when inference is arrived at from probable and authoritative statements it is called rhetorical, and when it uses false statements it is called fallacy. Thus demonstration is the only inference that is certain and always true. If we examine the principles of inference, referred to above, we shall find that most of them are not really principles but derivatives.

For example generally acceptable statements, considered by formal logic [???] principles of inference, may be regarded as a starting point in a discussion between two persons; but they are not real principles of thought. Further, authoritative statements are also derivatives because regarding a statement as trustworthy on the basis of divinity of otherwise means deriving it from other statements based on divinity. And probable statements usually used by formal logic are really derived from other statements which are probable not certain. For example, in the inference 'this [piece of iron] extends by heat because it is metal and all metal extends by heat', 'this extends by heat' is certain though derived statement, and 'all metal extends by heat' is empirical and included under the six classes of certain statements, already given.

On the other hand, in the inference 'this person is rude because he has no job and nine of every jobless ten are rude', this is [???] rude' has 9/10 probability, and nine of every [... ???] rude' also empirical[???]. Now, difference between the two examples is that the former includes certain premises, while the latter does not. Finally illusive statements are in fact inductive, though the generalisation is false. We may now conclude that the six classes of certain statements are the ultimate principles of knowledge, and all other statements are derived from them; if these are logically derived they are also certain, but if they are mistakenly derived they become false or illusive.

In what follows, we shall discuss this theory of the sources of knowledge, adduced by formal logic. We shall ask the following questions. Is it valid to consider universally empirical, intuitive, testimonial and basic empirical statements as primitive? What are the limits of human knowledge if our interpretation of inductive inference is accepted? Is there is any *a priori* knowledge? Can knowledge have a beginning? And finally can primitive knowledge be necessarily certain?

Universal empirical statements

We have shown that universal empirical statements, for formal logicians, are among the classes of basic statements, though they are logically preceded in order of time by empirical statements. For we usually begin with such statements as 'this piece of iron extends by heat', and proceed to 'all iron extends by heat'. But formal logic in its classification of propositions does not consider universal empirical statements as inferred from basic empirical statements. For the former have more than the total of the latter by virtue of the process of generalisation.

Thus, when formal logic classifies statements into primary[/primitive???] and secondary, and includes universal statements among secondary ones, it regards them as derived from an important primary statement, namely, relative chance cannot prevail. Accordingly, on observing the uniform relation between the extension of iron and heat, we may infer that heat causes extension. For if this occurred by chance, we would have not observed the uniform relation. The basic statement would be 'relative chance cannot permanently recur' and such statement as 'all iron extends by heat' as inferred. Thus formal logic gives two different claims, namely, universal statements are basic, and they are inferred from the statement denying chance. And we have already argued that the latter claim, is not basic and independent of experience but it is derived from experience. This does not mean to deny that such statement could be a ground of empirical statements in latter stages of empirical thinking. That is, if we can empirically verify the statement 'relative chance cannot permanently recur', we may deduce from it other empirical statements.

But if we take empirical statements as a whole, we cannot take such statement as ground of them all. Thus formal logic in this is defective. Again, it is false to agree with formal logicians in claiming that empirical statements are primitive not derived from other inductive statements.

To make our criticism clear, we may first distinguish between two concepts of the relation between an empirical statement such as all iron extend by heat and particular statements such as this piece of iron extends by heat.

Any particular statement of this kind expresses only one particular case of a general statement, thus this latter contains more than what is conveyed by particular statements. But we may also regard a particular statement involving the whole content completely. Thus general statements are derivative in this sense. Accordingly, derivative empirical statements are three classes. First, particular statements which constitute general statement inductively. Second, the postulates required for inductive inference in its deductive phase, since these postulates are the ground for confirming any statement of the first class. And we have already seen that these postulates satisfy the *a priori* probability of causality on rationalistic lines. The third class contains the postulates required for probability theory in general, for determining degrees of credulity.

We may remark that inference from empirical statements is probable not certain. Hence any empirical statement is derived so for[???] as certainty is concerned, whereas certainty involved in empirical statements is not logically derived from other statements, but it is a result of multitudes of probabilities.

Intuitive statements

Intuitive statements are similar to universal empirical ones. An example of the former is 'the moon differs in shape according to its distance from the sun'; we intuitively know that the moon derives its light from the sun, in the same way that we know that heat is the cause of the extension of iron, owing to observing the concomitance between heat and extension. Formal logic considers intuitive statements as primary, but it considers them statement which is the ground of empirical statements, namely, that relative chance does not permanently recur. For unless the moon derives its light from the sun, the difference in the distance between them would not have been connected with the various forms of the moon.

Now, we take it that intuitive statements are inferred from particular statements constituting their general form. But intuitive statements are not certain. Certainty adduced to these statements is merely a degree of credulity. That is, we cannot confirm it by means of prior statements, but we cannot at the same time obtain such certainty except as an outcome of probabilities. Thus certainty attributed to empirical and intuitive statements presupposes prior statements, though not deduced from them.

Testimonial statements

This is the third class of certain statements for formal logic, for our belief in the persons or events we are told to exist is primary. This means that formal logic postulates that a great number of people cannot give lies, and this [is] similar to the postulate 'chance cannot permanently occur'. Thus giving lies cannot always occur.

Suppose a number of people have met in a ceremony, and asked each other who was the lecturer, and suppose all answers referred to one and the same person, therefore we say that the answer expresses a testimonial statement. Our belief in such statement is really based on induction not on reason. Testimonial statements are really inductive and based on inductive premises. Those statements are concerned with the second form of inductive inference. We have previously shown that induction has two forms, the first is concerned with proving that a causes [b???] though we know nothing of the essence of both. The second form of induction is concerned with the existence of [a???] and its being simultaneous with b, knowing that a causes b, but we doubt the existence of a. This form involves the question whether the cause of b is c or d. Testimonial statements deal with such sort of induction. For example, if a group of persons agreed on the name of the lecturer, here the latter is a and the various answers of persons are b. The alternative for a is to suppose that all persons have given a lie for some reasons. This enables us to form an indefinite knowledge containing probabilities about such reasons. These will be eight if we have 3 persons. We may have the probability that only one person has a personal interest in lying, or the probability that two have interest in lying, or the three, or else that such interest is absent in all.

Each probability involves three suppositions, thus the sum of supposition in this knowledge is eight assuming that we have three persons. Seven of those suppositions imply that at least one person has no interest in the lie, and the eighth, implying that all have personal interest in lying, is indifferent as to the truth or falsehood of the statement.

If the value of having the personal interest in the news given by each person is 1/2, then having three persons, the value would be 7.5/8 = 15/16 included in the indefinite knowledge of a; and if we have four persons the value rises to (15+ 1/2)/16 = 31/32, until we reach the value of a very small fraction in case of denying the statement expressed by the answer given. Then begins the second step of inductive inference where the small fraction is neglected and is transformed into certainty. For the necessary condition of the second step of induction is fulfilled, namely, the neglect of the small fraction of probability value contrary to fact does not rule out one of the equal values.

This condition is made clear as follows.

(1) knowledge which embraces all possible cases of supposing personal interests in giving news in the source of probability values on a certain matter, and such values supersede the value of the contrary probability. (2???) It is observed in this connexion that the non-occurrence of an event is not included in such indefinite knowledge, it is rather necessary in this knowledge, because it is the case which involves the supposition of the personal interest concerned. The non-occurrence of an event does not apply except in this case. We have already shown in considering the second phase of inductive inference

that knowledge in such phase affords superseding one of its items.

Probability values of items are unequal within the knowledge embracing possible cases of assuming personal interests. This means that such knowledge affords superseding the probability of one of its items, without superseding other equal values. The reason why the values of items are unequal is that the value of the case assuming personal interest in informing news is smaller than that of any other probability, because the probability of recurring chances uniformly is smaller than other probabilities. If you try to throw a piece of coin ten times, the probability that it appears on its head or its tail all the times is less than any other; likewise, in testimonial statements, the case of there being personal interests in giving news about an event is less probable than any other case.

(3) We have explained this by introducing another indefinite knowledge in which this case has less value than other cases. The persons concerned have different circumstances and their difference are far more numerous than their agreements. And supposing the agreement among all testimonies in those circumstances resulting in the personal motive for the news, such supposition means that it is items of agreement which determine the judgment of all testimonies; and this makes the probability of the uniform recurrence of chance less effective than the other probability. In consequence, the indefinite knowledge involving possible cases of supposing

the personal motive for giving certain news does not include equal terms of probability value, because the value of their being a personal motive of information is the intrusion of another indefinite knowledge. Thus, indefinite knowledge may possibly supersede the probability value of such personal motive, without leading to the ruling out of one of its equal values. Accordingly, we can distinguish testimonies agreeing on a certain matter form those which disagree. When there is complete agreement on some fact, the belief that at least one person gives us the true news is more trustworthy than the case in which each person of a group gives different information. Testimonial statements are then inductive inference deals with any inductive statement, in two stages, namely, the calculus of probability and the grouping of probability values toward one direction.

Testimonial statements and *a priori* probability

These statements give rise to a problem concerned with *a priori* probability, to which we may turn. Although indefinite knowledge deluding the probabilities of truth and falsity issues the grouping large values cannot determine the ultimate value of testimonial statement. But we may here consider the *a priori* probability of this statement derived from prior knowledge, in order to determine the ultimate value by multiplying one knowledge in another.

For example, suppose we have a piece of paper on which are written words containing a hundred letters, but we know nothing more about such words. We have then a great number of *a priori* probabilities because there are 28 probabilities in each of the 100 letters, thus the sum of possible probabilities is the product of 28 in itself hundred times. And this is a fabulous number constituting an indefinite knowledge, let us call it '*a priori* indefinite knowledge". If hundred men inform us of a definite form of those various forms of words and that each man in his information is moved with a personal interest with the probability 1/2, then we get an indefinite knowledge of the possible forms of the being or absence of personal interests, such knowledge may by called *a posteriori* indefinite knowledge'. The number of such forms is 2 X 2 hundred times.

For each man has in his information two equal probabilities, namely, that he may or may not have personal motive, and by multiplying the two probabilities, for each man we obtain a great number of possible forms. All these forms, except one, involve that at least one of the hundred men has no personal motive, and means that the testimonial statement is true. But this exceptional form is indifferent. When we compare the probability value depending on the knowledge expressing the testimonial statement with the value depending on the *a priori* knowledge denying that statement, we find that the latter value is larger than the former. For the favourable value depends on the grouping of the values of the items of the *a* *posteriori* knowledge, with the exception of half value of one item, and it is the truth of the testimonial statement. And the unfavourable value depends on the grouping of the values of the items of *a priori* knowledge. The number of the items of this latter knowledge is much greater than those of *a posteriori* knowledge, because the items or the *a priori* knowledge are equal to the multiplication of the 28 letters in themselves hundred times, while the items of *a posteriori* knowledge are equal to the multiplication of 2 in itself hundred times.

And this means that the probability value of the testimonial statement cannot be large enough, thus inductive inference in the way stated hitherto cannot explain testimonies.

Solution of the Problem

This problem can be solved with an application of the third additive postulate (the dominance postulate) instead of the postulate of inverse probability, because the probability value favouring a testimonial statement dominates the value inconsistent with it. For the *a priori* knowledge is concerned with something universal, i.e. one of the possible construction of the hundred letters. We know that the actual form of letters on the paper is that for which there is no personal motive, and this is the content of the *a priori* knowledge. Now, if we look at any value involving that at least one of the 100 information has no personal motive, such value is inconsistent with the truth of any other combination of words contained in the *a priori* knowledge. This proves that the value favouring the testimonial statement dominates the value contrary to it, and thus the faintness of the *a priori* probability value of testimonials cannot hinder inductive inference.

But the faintness of the *a priori* probability of testimonial statements cannot be an obstacle to induction if this faintness arises out of various alternatives to testimonial statements, such as we have seen in the last example, that the actual combination of words of which there is a complete consent as one of the great number of possible combinations. In such a case the probability value derived from the *a posteriori* knowledge favouring the testimonial statement, dominates the value derived from *a priori* knowledge denying the statement.

On the other hand, if the faintness of *a priori* probability of a statement depends, not on the multitude of alternatives, but on probability calculus in the stage of giving a reason for this testimonial statement, the faint value will have a positive role hindering inductive inference. For example, suppose an Arab write on a piece of paper hundred letters, and informs many persons that he has written hundred letters in Chinese. Then we notice that *a priori* probability of writing hundred letter in Chinese is very small, the cause writing of hundred Chinese letters depends on knowing Chinese which is not familiar among Arabs.

Suppose that in every ten million Arabs, there is our knowing Chinese; this means that the probability of knowing that someone knows Chinese among that number of men is one - ten millionth, and that there are ten million probabilities constituting an indefinite knowledge. The largest value in this knowledge denies that x knows Chinese; in consequence, there arises a large negative probability value of x's knowing Chinese. In such a case we obtain three kinds of indefinite knowledge: (a) the knowledge that the writer writes either Chinese or Arabic, (b) the knowledge that there are people saying that he wrote Chinese letters, that the items of such indefinite knowledge is the product of 2 in itself as times as the number of the people giving testimony, provided that the probability of there being or not being a personal motive is 1/2; (c) the indefinite knowledge that the person writing Chinese letters is one of the ten million people, that it has ten million items one of which involves knowing Chinese while others involve ignorance of Chinese.

If we take notice of the value that x wrote Chinese letters on the ground of the first knowledge, we see that it is 1/2, provided we have only two languages. But if we look at the value within the second knowledge, we find it very large, because most of the values here deny any personal motive by testimonies. Again, the probability value within the third knowledge is found very small, because most of the values here deny that x knows

Chinese, and this means that the value depending on the first knowledge mediates two inverse attractions.

We have already stated that the large probability value, affirming testimonial statements derived from the second indefinite knowledge, dominates the value denying that statement derived from the first indefinite knowledge, we similarly claim that the large value denying testimonial statements and derived from the third indefinite knowledge dominates the value affirming them and derived from the first knowledge.

In order to confirm such dominance, we say that the first indefinite knowledge is concerned with a restricted universal, namely, that the author wrote a language known to him. The large value denying the testified statement and derived from the third knowledge denies that the author knows Chinese, thus it denies the fact of Chinese script. In consequence, the probability value affirming the Chinese script and derived from the first knowledge is dominated by the probability value denying that the writer knows Chinese which is derived from the third indefinite knowledge. And the value denying such knowledge is dominated by the value derived from the second indefinite knowledge; the former value assures that at least one testimony is not based on a personal motive. Therefore appears the positive role played by the *a priori* probability.

But if we do not know yet that x knows Arabic, only we know that x knows either Arabic or

Chinese, and that the probability of his knowing Chinese is one ten millionth according to the third knowledge, then it is impossible to explain the dominance of the value, derived from the third knowledge, on the value derived from the first knowledge on the basis of third additional postulate. For in such a case both values give rise to the denial of the restricted universal belonging to the other knowledge, namely, the writing of a language which the writer knows.

Whereas the restricted universal belonging to the third knowledge is that the writer knows the language written on a paper. The value derived from the third knowledge, denying the knowledge of Chinese script is inconsistent with the universal belonging to the first knowledge.

This position can be attacked with the help of the fourth additional postulate which says that real, not artificial, restriction produces dominance. This letter postulate states that the probability value determined by indefinite knowledge of causes, dominates the value determined by knowledge of effects. The case with which we are now concerned is one to which the fourth postulate applies, because the third indefinite knowledge is that of causes and the first knowledge is that of effects.

Belief in rational agent

We usually believe that other men, whom we know, have minds and thought. When we read a book consistently written for example, we believe that its author is a rational being, and deny the probability that he is irrational or lunatic and that such book is produced by mere chance.

It may be claimed by someone, who thinks on Aristotelian lines, that inferring that such author has a mind is inference from effect to cause. Indeed, the book is an effect produced by some author, but such book does not logically prove that the author is a rational thinking being. It may be so, but it may be also that the author is a lunatic having some random ideas which constitute the book. In both cases the principle of causality is at work. Inductive inference is a basis of the first probability but not the second. For the second supposition involves many particular suppositions according to the number of the contents of the book. In such a supposition, there is no connection or consistency among the successive contents of the book; and this means that this second supposition cannot explain the rational production of the book.

On the other hand, the first supposition involves that ideas expressed in the book are connected and systematically related to each other. Suppose the word boiling occurred in the book hundred times, defined, explained and exemplified in the relevant way. This explains that the author has understood that word, and that he is a thinking being. In consequence, two sorts of indefinite knowledge arise. First, the knowledge which includes the probabilities required of the first supposition, suppose we have three ideas a, b, and c; here we have eight probabilities as to their truth and falsity.

It may happen that (a) only or (b) only or (c) only is true, or all are true.

Such indefinite knowledge denies the first supposition with a great probability. For all its items, except the one in which all ideas are true, deny the first supposition. The exceptional case will be indifferent, because if a, b and c are all true, they may be so as a result of rational process or of chance.

The second indefinite knowledge includes the probabilities required of the second supposition, and since the latter is more complex than the first supposition, its items are much more than this. This second knowledge denies the second supposition with a greater probability value than the value given by the first knowledge to deny the first supposition. But the two negative values are incoherent because one of the suppositions in fact occurs. Thus we must determine the total value by means of the multiplication rules and here we get a third indefinite knowledge which embraces all possible probabilities. In this last knowledge the negative value of the second supposition will be very large. And this application of induction belongs to the first of the cases of the second forms of induction.

Inductive proof of God's existence

Instead of the example of the book, we may now suppose as object of induction, a group of physical phenomena. We may use inductive inference to conclude that such phenomena have a wise Maker. When we consider the conceivable hypotheses relevant to explain a group of phenomena, such as those of which the physiological composition of a particular man consists, we might have before us the following hypotheses:

(1) explaining those phenomena by virtue of a wise Creator,

(2) or by mere chance,

(3) or by virtue of an unwise maker having non-purposive actions

(4) or by means of non-purposive causal relations produced by matter.

What we hope to show is to verify the first hypothesis and refute the other ones. To accomplish this and, we offer the following points.

1. We must know to begin with low[???] to determine the value of the *a priori* probability of the hypothesis in question, that is, what is the probability value of there being a wise Creator having the required consciousness and knowledge for when we obtain an *a posteriori* indefinite knowledge increasing this probability inductively, we can compare the value of *a priori* probability and that of *a posteriori* probability, and by multiplication we come to the required value.

We need to suppose certain opinions to defend the hypothesis that the physiological composition of Socrates for instance is due to a wise Maker. Any of these opinions is regarded as elements in the hypothesis in question, and its value may be determined *a priori* by 1/2. For the being or non-being of such element is involved in the second additional postulate, this we obtain an indefinite knowledge having two members, the value of each of which is half, and this value is not refuted by introducing causes or effects. Now, if the value of each element of hypothesis is 1/2, then the value of all the elements is 1/2 multiplied in the number of elements. This value is included in an indefinite knowledge, different from the first, let it be called knowledge. Thus, we get an idea about evaluating the *a priori* probability of the hypothesis in question. But it is difficult to determine its value, because we do not know the number of elements of the hypothesis thus we cannot know the number of elements included in knowledge 1.

2. Suppose for the moment, that we confine ourselves to Socrates' physiological constitution within two hypotheses only, namely, that it is due to a wise Maker or to absolute chance. Now, we want to get an indefinite knowledge determining the value of *a posteriori* probability of the first hypothesis, let that knowledge be knowledge 1.

This is formulated thus: if there were no wise Being creating Socrates, the non-existence of Socrates would have been probable, or Socrates would have been existed in any other way consistent with the way he in fact is. All probabilities of the consequent, except the last, refutes the antecedent, thus we deny this latter, that is, affirm the first hypothesis.

3. In order to determine the total value of the probability of the first hypothesis, we have to

multiply the number of items of knowledge 1 in those of knowledge 1, and subtract the improbable cases. But here we have before us the problem, that we do not know yet all the items of sorts of knowledge.

4. In consequence, we have to offer a rule which enables us to get the value of the probability of any items in that knowledge, the items of which we do not know. But since we do not know this, the value of the fraction cannot be determined. However, we can get the approximate value if we follow the following points.

First, if we have two sorts of indefinite knowledge the elements of which we do not know, and if the probability of the number of elements in one knowledge is equal to that in the other, then the number of elements in each is equal to that in the other. That is the actual value of one element in the one knowledge is equal to the actual value of one element in the other, and the value of the element belonging to a knowledge, the number of the elements of which we do not know, may be determined in the following way.

We assume that n_2 is the probability value that the items of the indefinite knowledge are two, that n_3 is the value that the number of items are three, and so on. We also assume that x_2 is the value of the one item supposing n_2 and x_3 is the value of the one item supposing n3, and so on. Thus we determine the value of this element thus: $n_2x_2 + n_3x_3 + n_4x_4 + ...$

When we clearly face two sorts of indefinite knowledge in the way aforementioned, the process determining the value of an item in each knowledge will be similar to that which determines the value of a item in the other knowledge. Therefore their values are equal. This means that the value of the denial of a determined item in one knowledge is at the same time the same value of the denial of a determined item in the other.

Secondly, whenever we have two sorts of indefinite knowledge (let us say a, b), [and] the number of their members is unknown except that a is larger than b, and whenever we have two other sorts of indefinite knowledge (c, d), but we know only that c is larger in number than d, here we have four indefinite knowledge the number of their members we know only that a is larger than b and c larger than d. In such a situation, the actual value of a equals that of c, and that of b is equal to that of d. This means that the value of a member of a is less than the value of a member of d, still less than a member in b which we already know to be less than a. On the other hand, the value of the denial of one member in a is larger than the value of denying another member in d. For all probabilities assuming that the members of a are not less than the members of c, show that the members of (a) are larger than those in (d), since there is a chance that members of (a) may be more than those of (d) while there is no contrary chance that members of (d) are more than those of (a).

Thirdly, assuming that we have four kinds of indefinite knowledge a, b, c, and d; that we do not know the number of items in each, but we only know that items in (a) are larger than those in (b), that those in (c) are more than those in (d), that we also know that the ratio of increase in the former is more than the latter- in such a case (a) would be more in the number of items than (c), in the sense that the value of the one item in (a) is less than that in (c), and that the inverse value of the one item in (a) is larger than that in (c). For all not exceed (b) entail that (a) is larger than (c). Whereas the probabilities implying that (d) exceeds (b) do not entail that (c) is larger than (a). Thus there are probability values denoting that (a) is larger than (c), but there is none denoting the contrary.

Fourthly, if we keep (a), (b), and (d), and know that (a) has more members than (b), but know nothing about (d), and do not assume (c), then (a) has more members than (d), because all probabilities implying that (d) does not exceed (b) entail that (a) has more members than (d). But the probabilities implying that (d) exceeds (b) do not entail the converse. In consequence, the value of a member in (a) is less than that of a member in (d), and the value of denying a member in (a) is larger than that in (d) All these statement form a rule for the relative determination of the value of a member belonging to acknowledge the members of which we do not know.

Fifthly, in view of what has been said, we may suppose that the number of members of knowledge 1 and that of knowledge 1[???] is identical. That is, knowledge which includes all the elements of the hypothesis of a Supreme Being, is equal in its value to the knowledge which includes all the elements of the hypothesis of chance. For we have no idea of the number of elements in each. It follows that knowledge 1 provides a favourable value to refuting the first hypothesis, and that knowledge 1 provides favourable value to such hypothesis. And if we assume the two hypotheses to be equal then any multiplication would also give equal values.

But Socrates is not the only human being, but there might be Smith for example who owns a set of phenomena to be explained in terms of each of these two hypotheses; thus we obtain knowledge 2 and knowledge 2. We may construct another indefinite knowledge having more members than knowledge 1 and knowledge 2, which is a product of the members of both, let this new sort of knowledge be knowledge 3. Now, if this has more members than the others, then the value of the probability of absolute chance is much less than the value of the probability of absolute chance belonging to Socrates or Smith alone. And since knowledge 3 has more members than knowledge 2 and knowledge 1, it is also larger than knowledge 1 and knowledge 2, because knowledge 3 represents (a), knowledge 1 and knowledge 2 represent (b), and knowledge 1 and knowledge 2 represent (d). As we obtain the indefinite knowledge 3, we can obtain knowledge 3, which determines the probability value of a Supreme Maker of Socrates and Smith. But this knowledge has no more members than those in knowledge 1 or knowledge 2, because the elements of the hypothesis of a Maker of Socrates are the same as those of a Maker of Smith.

Thus we have before us six sorts of indefinite knowledge: knowledge, knowledge 2, knowledge 3, knowledge 1, knowledge 2, knowledge 3[???]. We do not know, we only know that the members of knowledge 3 exceed those of knowledge 1 or knowledge 2, and that the ratio of excess in the former is larger than the latter. Thus, we may argue that knowledge 3 has more may argue that knowledge 3 has more members than knowledge 3, because knowledge 3 represents (a), knowledge 3 represents (c), knowledge 1 and knowledge 2 represent (b), and knowledge 1 and knowledge 2 represent (d). But we have already argued that (a) has more members than (c) and this means that the value of a member in knowledge 3 is less than that of a member in knowledge 3 and that the value of denying a member in knowledge 3 is larger than the value of denying a member in knowledge 3.

Since the value of denying a member in knowledge 3 is larger than the value of denying a member in knowledge 3, then the value of refuting the second hypothesis is larger than the value of refuting the first hypothesis. And when knowledge 3 and knowledge 3 are multiplied and a third indefinite knowledge is obtained to determine our values, then the value of refuting the second hypothesis will be much larger than the value of refuting the first hypothesis. Thus the number of factors refuting the first and second hypothesis is constant in the third indefinite knowledge.

And since we know that knowledge 3 has more members then knowledge 3[2???], the value of the probability of the first hypothesis derived from the third knowledge is necessarily much larger than the value of the probability of the second hypothesis derived from this knowledge. Therefore, the probability of the first hypothesis increases in value.

Sixthly, likewise, we can explain the developing value of the first hypothesis in opposition to the third hypothesis supposing phenomena to be result of irrational being. If we supply additional phenomena we get a new knowledge, and consequently a new knowledge 3. Here, we find that the value of the probability of there being an irrational entity, producing all phenomena, will be very faint, whereas the value of the probability of the Wise Being hypothesis will not be so. For supposing an irrational being producing all phenomena implies new numerous elements not involved in the first hypothesis.

There is a final hypothesis stating that Socrates' physiological constitution is explained by some causal relations among such constitution and other phenomena.

We have to re-formulate this hypothesis in order to falsify it. For if we add Smith [???] for example, Socrates, we do not obtain more elements in the hypothesis because causal relations are connotational dealing with essence. If we assume that the matter of which Socrates is composed entails his physiological constitution, this means that there is a similar relation between Smith's matter and constitution.

Now, in order to get a new hypothesis we have to imagine a different kind of matter for each. Thus, we can construct on indefinite knowledge 3 which these later, except one, refute the fourth hypothesis, knowledge 3 does not include such number. Therefore, we may speak of the physiological constitution of sexual reproductive system in the male, and a different constitution in the female.

But, though they are different, they have something in common which could be explained only by supposing a Supreme Being.

Basic Empirical Statements

We have already mentioned the six-fold classification of statements from the Aristotelian point of view, and have now considered five of them. Basic empirical statements are the last class now to be considered. For formal logic, this class is first step to acquire human knowledge.

Basic empirical statements are divided into two kinds, what belongs to outer sense and what belongs to inner sense. "The sun is now rising' is an example of the former, 'I feel pain' is one of the latter. Basic empirical statements belonging to inner sense is doubtless basic because the appeal to inner private sense is the only test for its truth. Whereas statements belonging to outer sense involve the existence of an external world, and thus may be doubted. There are two formulae of such empirical statements, which justify us in doubting their certainty.

First, in our perception of lightning, for example, our direct awareness of it does not itself enable us to claim that there is something external to us called lightning; perception itself does not enable us to distinguish subjective states from objective reality. My perception of lightning is a basic empirical fact but the being of lightning is not. Secondly, even if we could distinguish the subjective from the objective elements in perception, perception of an object is not the objective fact itself but a subjective event in our brain or mind. It may be said that such subjective event is causally related to an external object, but the latter is not itself revealed in perceptual situation itself. Both formulae denote one thing, namely, that objectivity or external reality is not an immediate given to sense, thus this reality is still to be argued for. Therefore, idealism denies the belief in the existence of external objects on the ground that our empirical knowledge does not justify this belief. However, we have noticed that Aristotelians claimed that the objectivity of the event perceived is involved in basic empirical knowledge.

The supposition of objective sensible reality is not without justification as idealism claims, not is it indubitable basic knowledge as Aristotelians have argued. Such supposition is got by inductive inference. For belief in objective reality is based on the grouping of probability value in a definite direction, and such grouping of values is transformed into certainty if certain conditions, hitherto stated, are fulfilled. In what follows, we give some of the inductive ways by which we arrive at basic empirical statements. We shall consider the two formulations of our doubts in objectivity separately.

Inductive ways concerning the first formulation

(1) Suppose I am in a situation in which I perceive lightning and under [???], and do not know yet whether they are merely subjective states of the mind or also refer to a physical fact. I have a doubt not in the perceptual fact, but in interpreting it, whether it is caused by me or has an external cause. Now, both hypotheses are equally probable. This means that the value of the proposition 'the occurrence of lightning is an objective fact[5] is equal in its probability to the proposition that my perception is a subjective event. Both probabilities are represented as items in an indefinite knowledge which we may determine the *a priori* probability that the event in question is objective.

(2) Before considering whether the event is subjective or objective, we must maintain the

principle of causality inductively, and this we have given in detail in the previous chapter.

(3) Very often and through uniform observation, we see certain [???lings] succeeding each, such as light and sunrise, thunder and lightning, boiling and heat etc. Such permanent concomitance may be taken as ground of causal relations between any items.

(4) It often happens that we perceive the effect (b) without perceiving its cause (a) which is already known by induction to be prior. We may see light without seeing the sun, hear thunderstorm without seeing lightning. In such situations, we have a cause which would be (a) known inductively to be its cause, or any other unknown event, (c) for instance. This case is represented in another indefinite knowledge called 'second *a priori* indefinite knowledge'. In the latter, the objectivity of (a) is involved since its existence is assumed though unperceived.

(5) Added to the first and second *a priori* indefinite knowledge, here is '*a posteriori* indefinite knowledge which determines that (c) is not cause of (b) because (c) as not cause is only a probability not certainty, and being probable there be an indefinite knowledge, does not only deny that c is cause, but affirms something - since (a) occurs in the vicinity of (b) and (a) is known inductively to be cause of (b), thus we get the affirmation that (a) is the cause of (b); and a fortiori (a) as objective. We now conclude that the probability that (c) is not a cause

is inconsistent with the probability that the event (b) is merely a subjective mental state.

(6) When we compare the improbability of (c) as cause with the second *a priori* knowledge, we notice that the former dominates the value determined by the latter. For such improbability denies any essential relation between (b) and (c). And this involves the exceeding probability that (a) is the cause of (b).

(7) When we compare the improbability of (c) as cause with the first *a priori* knowledge, we find that such improbability supersedes one of the hypotheses of that knowledge namely, the subjectivity of (a).

(8) The outcome is that we obtain probability values increasing the objectivity of the event in question-such values deny that (c) is a cause of (b) and determine the degree of increase of objectivity if we multiply the items of *a posteriori* knowledge and the first *a priori* knowledge. And the more cases we have the larger the value we get for the objectivity of events.

Inductive ways concerning the second formulation

(9)In the previous paragraphs, we have applied inductive inference to basic empirical statements within the first formulation of the possibility of doubt in the truth of those statements, wherein we arrived at the objectivity of the events (a) and (b). Now, we consider the second formulation of the possibility of doubt in those basic statements. This formulation is that the event (b) is a felt subjective state of the mind, and we have no indefinite knowledge as to whether it is really subjective or has it objective reference. In what follows we give the inductive steps by which we establish those basic empirical statements.

First, when we perceive the event (b) without perceiving the event (a), for example when we hear thunder without seeing lightning, we have an indefinite knowledge that (b), being surely subjective, is either caused by an objective fact (a)[x(b)] or caused by another subjective event (c).

Secondly, if our perception of (b) is caused by another subjective event (c), this latter in turn requires a hypothesis to explain it, namely, that (c) has a cause (d). Or we may suppose that the subjective event is caused by an objective fact (a) [x(b)]. Now, even if the latter requires another fact[xpact] as causing it, it is still valid to suppose that the subjective event (b) is caused by an objective fact. Such concomitance between the subjective [xobjective] (b) and the objective (a) is constant; thus the inductive argument that the objective fact (a) is cause of the subjective [xobjective] fact (b). Whereas the hypothesis that the subjective event (b) is caused by another subjective event (c) is not constant, hence, the increase of the probability that our subjective events require objective reference outside our minds.

Thirdly, there is [xhere] a point implicitly assumed [here], namely, all events are either caused by objective events or by subjective events. Thus if we regard (b) being subjective, as caused by an objective fact (a) [x(b)], we are considering the first hypothesis stating the regular concomitance between subjective [xobjective] (b) and objective (a). But there is nothing to justify this point - nothing prevents supposing that (b) subjectively given is not preceded by (a) also caused by the objective fact (a) [x(b)].

Fourthly, it is possible to increase the probability of objectivity if we assume the objectivity of all events or the subjectivity of them all, thus the value of absolute objectivity exceeds the value of absolute subjectivity.

However, this is insufficient to increase the probability of objectivity in a single event. To overcome this difficulty, we may try the following formulation. When we have in our experience subjective [???] and (b), we obtain an indefinite knowledge the (b) is caused either by a subjective (c) [x(a)] or objective (a) [x(b)]; and when we perceive the subjective event (b) without the subjective event (c) [x(a)], we obtain another indefinite knowledge that the subjective (b) is caused either by an objective (a) [x(b)] or another subjective event (c). This means that the objective (b???) causing the subjective (b) is common in that two kinds of knowledge, and this increases the probability of causality between the objective (b???) and the subjective (b) much more than the causality between the subjective (b) and the subjective (c). Therefore, the probability of the objective explanation of the subjective event (b) exceeds that of explaining it subjectively in relation to (c). For, the former implies there being a causal relation between the objective (???) and the subjective (b). Whereas the explanation of (b) in relation to (c) implies a causal relation between the subjective (b) and the subjective (c). And since the first hypothesis is more probable than the second, then the probability of objectivity exceeds that of subjectivity.

(10) We may confirm the previous point with an argument from constancy, namely, when we abstain from perceiving a certain situation, and return to it, we perceive the same. This we make clear in the following points.

First, if we suppose that the perception of an object is purely subjective state of mind, then the probability that the perceived object recurs is very faint. For, the subjective situation ceases to exist after it has been recognised, and we may perceive a different object in the next moment. Thus we may claim that when the object is purely subjective it can never recur exactly as before.

Secondly, if we claim that involves a real object external to us, this is more probable on account of similar recurrence of the same object.

Thirdly, when we perceive the previous object again, and realise it to be almost the same, we may argue that this object of perception is objectively real, on the assumption that if it is not so it could not have been the same. And since the consequent is false, the antecedent is also false; thus the objectivity of perceived objects.

However, such objectivity depends on the probability value of the sameness of the object in the two successive instants, and that it is greater than the difference between them. For if both values are equal in force, then the truth of the matter between subjectivity and objectivity is indifferent. Now, if we suppose that the object perceived is a subjective state, then it is caused by something also subjective; and in the next experience, very probably I will perceive something different, thus the probability of subjectivity is very faint. On the other hand, if we suppose the object of perception to be objective in character, this involves that there is something in common among the object on successive intervals, and the permanence of such common characters is a probability but higher in value.

It may be argued that in supposing the objective character of the object of perception we may have more than two probabilities. Suppose we are looking at a friend, John for example; then when we see all his body in the normal way on two intervals in the same manner, we can say that the object of seeing is objective. If we start assuming that the object is a subjective state, then we may see John with one arm or three arms, or other probabilities. But in this case we naturally say for example, that the lost arm is broken, or the third arm is unusual scene. And this indicates that to be objective, an object must remain the same on successive situations.

Our knowledge of the external world is inductive

In view of what has passed, we are entitled to claim that our belief in the external world depends on induction, because 'the external world' means that we can entertain statements which involves a reality outside our perception of it. And we have just argued that belief in the objective reality of perceptual statements is inferred inductively, that is, our knowledge of the world is accumulation of various beliefs in the objective reality of empirical statements. Thus the inductions confirming the objectivity of those statements assure us of objective reality. Consequently, we may face idealism which denies any justification for believing in the physical world, because this belief is inductively justified. Further, the common sense view of the world, being a reaction of idealism, is also answered, that is, this view which maintains that our knowledge of objective reality is so primary and immediate that it needs no inference. This commonsense view is answered by saying that at least some empirical statements are true owing to their probability values, and this explains their obviousness.

Belief in the conditions of perception is inductive

Our belief in the objective reference of empirical statements is the belief that when we get a sensible image of any object, and there exists certain conditions for its objectivity, then such object has objective reference.

But we possess further the belief that when we are confronted with an external object, we obtain a corresponding image, provided that certain conditions are fulfilled, and this other belief is inductively acquired. By conditions here we refer to the normal position the perceiver, the absence of a dim curtain, the normal quantity of light and the like. When these conditions are fulfilled uniformly or regularly, we inductively conclude that such relation between objective reality and those conditions has not randomly occurred, but that it is causal. If it happens that we do not get a certain image we infer that the object corresponding to it does not exist. For the absence of effect always denotes the absence of cause. When I believe that I am sitting in my study alone with nobody else, I assert a statement in the inductive way hitherto explained.

As it is an inductive matter to consider a physical object as cause of its image in my mind under certain conditions, it is likewise inductive to explain the occurrence of our ideas under these conditions. When I notice that a visual image in my experience gradually disappears, while another visual image of another object appears which seems nearer to me than the first object, we inductively conclude that whenever we have a visual percept of an object which seems nearer to us than another object, we lose sight of the latter, under certain conditions.

Thus our belief that we cannot see a person's hand put behind his back is inductive, that is, inferred from the fact that the hand is not seen under certain conditions.

Resemblance between percepts and realities

We usually believe in the resemblance, of a certain degree, between the sensible image of what we perceive and the object perceived. This belief is inductively acquired and not immediately given, because in our perception of the external world we have no immediate knowledge of physical objects, but we know the latter by the mediation of sensible images or percepts. When we see a square piece of wood, for instance, we are seeing an image, in our brain, having the property of squareness, and that it is an effect of the piece of wood really there. And we believe that this property noticed in our perception is also ascribed to the physical object. True, this point is usually owned by the commonsense view of the world but it is approved with lesser degrees by those who disclose more subjective factors in perception. However, it seems to be a minimum degree of resemblance between the percept and the object perceived. When we usually see round objects such as apples or oranges we do not usually ascribe to them squareness, though we have *a priori* basis of claiming that any physical object must cause a percept having the properties that it has; there is no self contradiction in saying there is a round object there causing in my mind a square-like shape. [this can be taken as a refutation to Kant's theory, which considers external world to get moulded into the constructions of space and time in mind, making the external objectivity a subjective (and distortion of the objectivity) in mind, refer to details in "Our Philosophy", reader's note]

Therefore, our belief in such resemblance is gained inductively, that is, what we see round in shape is really round. Suppose someone assumes that such roundish percept corresponds to a really square physical object. Here we have two alternatives: either we do not see the part of an object which faces us or see an object which is not really there. When we see a round sheet of paper and suppose it to be really square, then if this square is that kind of shape which can be drawn inside the round shape that we see, this implies that the spatial area of the image is bigger than the real sheet. If, conversely, this square is larger than what can be drawn inside the round shape we see, this implies that we do not see part of the square. Therefore, if I see all the parts and sides of the object then the percept resembles the real original.

When we have an *a priori* indefinite knowledge that the sheet of paper must have a shape, we know also that it actually has the specific shape that we see, if our perception has objective reference. And when we see the paper as round, we notice that the conditions being the objectivity of our perception, if it is regular, as the round shape is the specification of the *a priori* knowledge of the shape.

For, provided that our perception involves objective reference, the paper cannot be square. But if the condition is not fulfilled, that is, if our perception has no objective reference, then the paper probably has any other shape. So we conclude that the object of our indefinite knowledge is restricted to a hypothetical statement; and the restriction is that the paper is round if our perception is objective. But the latter fails, that is, the paper may have any shape.

Thus, any probability value affirming the antecedent affirms that the paper is round. We reach such value by induction. We notice the concomitance of seeing many sheets of paper with a certain shape, so we arrive at a greater value of the objectivity of our perception.

Beliefs in resemblances of particulars

We believe that certain things have something in common, so we call them a's, and other group of things having something is common, let us call them b's. Such belief is acquired inductively. We have already argued that there are resemblances between the physical objects and our perceptual images of them. When this happens we say that such many physical objects resemble each other. Resemblances among percepts are immediately recognised by us, but resemblances among things are inferred from resemblances of the former.

But our belief in resemblance between things and perceptual images is not itself sufficient for knowing resemblance among kinds of things, because we need suppose that nothing has changed in our sensory system and mental activities. For perceptual images depend on two factors, namely, the existence of the external world, on the one hand, and physical, physiological and psychological conditions of perception on the other hand. Now, if our sensory system is the same, we obtain the same images uniformly as before; but if the internal conditions have [underently???]. that is, it is probable that we get two similar percepts when there are two different objects before us, or two different percepts when the physical object is the same on two successive intervals.

In order to prove resemblance between any two physical objects on the basis of our perception of them, we have to obtain a significant probability value opposing any change in our subjective system. We recognise such resemblance by induction, and this process is necessary condition of proving that a is cause of b, (cause being a relation between two meanings), when we notice concomitance between a's and b's. In order to reach this conclusion, we must discover all a's to belong to one kind, all b's to another. Then we are able to increase the probability of causality between these kinds of things.

Recapitulation

We have so far explained four, out of the six, classes of statements, and concluded that all empirical, basic empirical, testimonial and intuitive statements are inductive depending on the accumulation of probabilities in a certain direction, according to two steps of inductive inference. We do not mean that certainty in these statements depends for every body on induction; belief in the objective reference of empirical statements depends for some people on confusing subjective and objective elements of perception. Again, belief in empirical statement may depend on purely psychological factors, or on expectation in terms of habit and conditioned reflex. What we mean is that our objective belief in those statements is based on induction.

Primitive and innate statements

These two classes of statements are considered by Aristotelians as certain, *a priori* and starting points of human knowledge; they are said to be apprehended independently of sense experience. Sense experience is introduced only, in relation to those statements, when we want to explain belief in them. Belief in these statements derives from conceiving their subjects and predicates, and experience supplies the mind with various images and meanings, these being the materials for conception. What can we say of this theory?

Primitive and innate statements, we claim, are inferred inductively. Let us explain. For Aristotelians, a primitive *a priori* statement is one the predicate of which is ascribed to its subject necessarily, that is, any subject of this kind implies a certain predicate. The statement 'the whole is larger than the part', 'or all rights angles are equal' are *a priori*, in the sense that the whole' implies being larger than any of its own part, or that if being right angle' is common among angles, this implies their being equal. But such implication relation between subject and predicate may be reached inductively, if we put forward two presuppositions: (a) the ascription of a predicate to a subject is a necessity for the latter, (b) such ascription is a consequence of an element different from the very notion of the subject.

Now, these presupposition are similar to those which we recognise when we observe (b) succeeding (a), and say either that (a) is cause of (b), or that (c) is cause of (b) while (a) and (c) are concomitant.

Now, if we suppose the implication relation between subject and [predicate??? precedent], we mean a relation between two concepts. But if we suppose that the ascription of predicate to subject is a result of a third element, then we have many probabilities before us in order to determine which thing is such element. In such a situation, inductive inference enables us to favour the first probability by the help of an indefinite knowledge which includes all forms of supposing what may be a cause of a certain predicate or property instead of triplication relation. For such indefinite knowledge gives us the accumulation of the probable values of that cause, thus affirming he implication relation between subject and predicate, save one value, i.e. that the only alternative to implication is involved in all cases when subject and predicate are correlated; and in this point, the exception is indifferent to all probabilities.

Therefore, we may affirm the implication relation inductively. And here we need not consider here the value of *a priori* probability of implication on the ground of indefinite knowledge prior to induction. For, the probability value of implication involved in the *a posteriori* knowledge dominates the *a priori* value of the probability of implication or its improbability.

Exceptions

There are two exceptions to inductive application of primitive statements, namely, the principle of non-contradiction and postulates of inductive inference itself.

First, the former states the impossibility of affirming two both contradictory statements. This cannot be proved by induction but must be presupposed *a priori*. For if we do not assume its truth beforehand, we cannot accumulate probable value in a certain problem, since such accumulation depends on the fact that any probability implies the negation of its contradictory.

Second, as to the postulates of inductive inference, any degree of approval of any statement is presupposed *a priori*. When we claim that it is possible to apply induction to all primitive statements, except the two principles, just referred to, we do not mean of course that such primitive statements are inductive not *a priori*, but we do mean that they can be theoretically explained in terms of inductive methods, though they may still he considered *a priori*.

Differences between primitive and inductive statements

One main difference between these two classes of statements is that more evidence would give inductive statement more truth, while more evidence adds nothing to the truth and clarity of *a priori* primitive statements. More inductive examples do not add to their truth because their truth is independent of experience, that is, it is *a priori* statement. The statement '1 + 1 = 2' does not become clearer or more true when we increase its relevant applications. Conversely, if we take the statement 'metals extend by heat', the more we collect instances of it, the more it is confirmed and its probability is established; thus it is inductive.

However, the difference is not easily made to distinguish *a priori* statements from empirical ones. For, there are empirical statements which become already settled by instances, so that new evidence does make no difference. For example, 'men who are beheaded die', or 'fire is hot' are inductive but they are so true and confirmed that they need no more confirmation. Thus, we do not consider such type of inductive statements in order to test their truth by looking for more evidence. In such cases we may not find the difference aforementioned relevant between a priori and empirical statements. Now, there is another difference between those types of statement, namely, if we have a feeling of the possibility of abandoning a statement, provided there is evidence against it, then this statement is empirical and inductive otherwise it is *a priori* and primitive. Suppose some trustworthy persons tell me that they have seen a beheaded [???] we probably believe them; nevertheless have a feeling of disbelieving them.

The third difference between *a priori* and empirical statements is that the latter cannot be always true in all possible worlds, however much we collect favourable evidence, and that they are true only in our sensible world. Whereas primitive a priori statements are absolutely true in our world and in any possible world. 'Tire is hot' is an empirical statement, for though it is obviously true in our world it is not necessarily true in any possible world, but we may intelligibly suppose a world in which there is cold fire. Whereas 'contradictory statements cannot be true together' is always true in any possible world we may imagine or conceive, because we cannot suppose a world where affirmation and negation or truth and falsity can be consistent with each other.

[Thus,] There are three differences between *a priori* or primitive and empirical or inductive statements, which are taken criteria for picking up primitive and innate statements.

Induction and mathematical statements

We have already shown that Aristotelian logic classifies statements into primary and secondary;

the former is in turn classified into six certain types of statements. This logic regards any inferred statement from these formal statements out of which our knowledge can be established. As such formal statements can be inferred from primary ones, they can also be inferred from those inferred from these. Thus formal statements derive from primary certainly true ones either directly or indirectly. The only way to infer the formal statements and prove them is through syllogism, the conclusion of which is implied in its premises.

We have made clear the relation of those six certain classes of statements to induction, and argued that they are inductive. Now, what relation there is between induction and formal statements, such as 'the angles of a triangle are equal to two right ones'. To give an answer to this question, we must distinguish the statements from they way it is inferred and whether the conclusion is validly inferred from its premises. As to the statement itself, it is no doubt possible to reach at [xI] it by induction not by deduction from *a priori* premises. Instead of inferring the previous example about triangle from the fundamental postulates of Euclidean geometry, it may be inferred inductively, in the same way in which the six classes of certain statements, discussed above are considered.

That is, we begin with assuming two probabilities:

(a) that a triangular figure implies that its angles equal two right ones,

(b) that such equality is due to an external reason (c). When we take notice of a triangle and observe that in every case, drawing it is always connected with the its three angles equalling 180°, we obtain an indefinite knowledge including the probability and improbability of (c), and we get a high probability value for the implication hypothesis. Such value derived from that knowledge dominates the value of *a priori* probability.

Further, as to the way in which our statements is inferred, it may be said that it is deduced *a priori* from certain postulates, but either the deducer is feeling sure that he proceeds validly, or that we may test his work on inductive grounds. First, the deducer may feel sure that he is right in choosing relevant postulates and proceeding rightly from premises to conclusions, and feels that conclusions follow from premises and that he does not fall in any mistake. Such feeling has nothing to do with induction but it is a personal direct feeling involving alertness and belief in the truth of its object.

Second, we may examine such deductive process inductively, that is, we may observe the number of mistakes the deducer has committed and then we determine the probability value of his falling into mistakes, and such determination is inductive. For the value is known through observing our activity and is considered as inductive inference showing that such value is not random but expresses the proportion of mistakes relative to right steps made.

Notes:

[18]This classification is definitely not Aristotle's; it may be medieval. And it is open to criticism, for example the 3rd and 4th classes are not among certain statements; the 5th cannot the intuitive (Trans.).

Chapter 2. Is There A priori Knowledge?

We have already referred to the main difference between rationalism and empiricism concerning the source and ground of knowledge. Philosophers through the ages have been divided into two classes[/groups] on this problem, those who believe that human knowledge involves an *a priori* element independently of experience, and those who believe that experience is the only source from which all sorts of knowledge spring, and that no *a priori* element is involved, even our knowledge of logic and mathematics. According to the former class (rationalists), man is believed to have some *a priori* ideas regarded as basis of our knowledge and stimulating our experience and explaining it.

But as the empiricists maintain that we have nothing a priori, that all knowledge derives from experience, through which everything is explained. In consequence, starting points of knowledge, for empiricism, are particular ideas supplied by experience, and all that this gives us is particular. But this situation involves that any general statement has more than what is particular, thus we are not justified in the certainty of such statements. Conversely, rationalism affords an explanation of this certainty on the ground of a priori ideas assumed.

Now, we must have before us a criterion by means of which we can compare and evaluate rationalism and empiricism. This criterion may be reached by pointing the minimal degree of belief in the truth of both formal and empirical statements. And any theory of knowledge disclaiming such criterion is doomed to failure, while it is approved when is consistent with such criterion. Now what is the minimal degree of credulity in formal and empirical statements?

Empirical Statements

This class of statement is given by rationalists a high degree of credulity reaching sometimes the degree of certainty while empiricism denies certainty to these statements since they depend on induction, but they acquire a higher degree of probability. Thus both theories of knowledge agree in regarding empirical statements as highly probable, and their probability increases by the increase of more instances. The question now arises, which of the two theories gives more satisfactory explanation than the other. We have already maintained that such degree of belief rests on the application of probability theory to induction. This theory has its own postulates some of which are mathematical in character. It is necessary then to regard such postulates as *a priori* statements independent of induction. This, we notice, is more consistent with rationalism than empiricism. For empiricism has to maintain, within its principles, that probability postulates are derived from experience; thus it has no basis for exceeding the values of probabilities. In short, empiricism cannot explain the minimal degree of belief in the truth of empirical statements.

Formal Statements

By these we mean in this context mathematical and logical statements. These have always made a problem for empiricists, in order to explain their certainty and the way they are distinguished from empirical statements. It is commonplace that a mathematical or logical statement is certain; thus if it is claimed that all knowledge derives from experience and induction, it follows that '2+2 = 4' or 'a straight line is the shortest distance between any two points' are inductive. If so, these statements are the same as empirical statements. Hence, empiricists have to choose either to ascribe certainty to formal statements only, or to make formal and empirical statements on the same footing. Either alternatives is a dilemma for them. For if they hold formal statements to be certain, then these cannot be inductive, it follows that we have to admit that they are *a priori*. And if formal statements derive from inexperience and not *a priori*, how can we explain their certainty?

The differences between empirical and formal statements are as follows. (1) Formal statements are so absolutely certain that they cannot conceivably be doubted, while empirical statements are not. Statements such as '1 +1 = 2' 'a triangle has three angles', or 'two is half of four' are very different from statements such as 'magnets attract iron', 'metals extend by heat', or 'men are mortal'.

The former cannot conceivably be doubted however sure we are about them. If we imagine someone we trust saying that there is water which does not boil when heated or that some metal does not extend by heat, we may possibly doubt general empirical statements. Whereas we cannot conceive denying such logical truth as 'two is half of four', even if the greatest number of men told us that two is not half four.

(2) More instances do not make mathematical statements more certain, while they do confirm empirical statements. When we supply more examples or new experiences about the expansion of metal by heat, we are more justified in claiming the truth of the statement. But if we observe only once that a magnet attracts iron, we have not established the truth of the statements unless we provide more and more and more instances. The case with formal statements is different, because when I can add five books to five others and realize that the sum in ten, then I judge that every two fives equal ten, whatever kinds of things I add, and the judgment is always true without giving more instances. In other words, once we hear or read a mathematical or logical statements and understand its meaning, we are sure of its absolute truth and certainty without the least of doubt; whereas the more we are supplied with instances that confirm an empirical statement, the more its truth is vindicated.

(3) Though general empirical statements are not confined to our actual observations and experiments, they concern our physical world and do not transcend it. For example, when we say that

water boils at a certain degree by heat, we transcend our actual observation but not our empirical world. But if we can conceive another world in which water boils at a different degree by heat, then we are not justified in making the general statement that water boils at that degree in that conceivable world. Conversely, mathematical and logical statements [admit???] of different consideration. The statement 2+2=4 is always true in any world we may conceive, and we cannot conceive a world in which a double two equal five; and this means that formal statements transcend the real in our sensible world.

Such differences between formal and empirical statements caused empiricism a dilemma in the way formal statements are to be explained, since to be consistent it has to explain then within its experience alone. Empiricism had to give formal statements a purely empirical explanation for some time, and thus made both kinds of statements on the same footing in that both are probably not certainly true. For empiricists, the statement 1+1=2 was probable and involved all the logical inadequacies ascribed to empirical generalisation. But this position proved empiricism to be on the wrong track and gave rationalism utmost credit, since the latter could explain the certainty of formal statements in terms of *a priori* knowledge and probability of empirical statements in terms of experience.

Logical Positivism

Empiricism has not changed its position in its empirical explanation of the truth of formal statement until the appearance of logical positivist movement in the present century[19]. Logical positivism admits the difference in nature between mathematical and logical statements on the one hand and empirical ones on the other. Such movement classifies mathematical statements into two classes, those of pure mathematics such as 1+1=2, and those of applied mathematics such as Euclidean postulates, e.g., any two straight lines intersect in only one point. The former are in essential, logical statements, and all logical and purely mathematical statements are necessary and certain, because they are tautologies.

The statement 2+2=4 does not give us information about anything empirical, but it is analytic. We may make clear this logical positivistic distinction between analytic and synthetic statements in some detail.

Synthetic or informative statements give us information about the world, in other words, the predicate in statements of this type is not included in the very meaning of the subject. 'Mortal' in the statement 'man is mortal', or Plato's [xTato's] teacher' in 'Socrates is Plato's teacher' is not part of the meaning of man or Socrates; so these statements give us new knowledge about men and Socrates. But analytic or tautological statements are those whose the predicate is part of the concept of the subject; the statement here gives us no new empirical knowledge but only analyses its subject. The bachelor is unmarried' is an example of analytic statements, because 'unmarried' is part of the meaning of ' bachelor'.

Now, logical positivists have tried to consider statements of pure mathematics and logic as analytic and explain their absolute certainty by means of their uninformative function. '1+1 = 2' is, for them, trying and sterile, because 2 is a sign identical to the signs (1 + 1), and then say that the two signs are identical. But statements of applied mathematics, e.g., postulates of Euclidean geometry, give us new information and knowledge. For positivists, 'straight line is the shortest distance between any two given points' is not analytic because shortness and distance are not part of the meaning of a straight line.

These statements are not considered by them necessary and *a priori*. "It was said about Euclidean geometry or any other deductive system that it deduces its theorems from certain axioms, these require no proof because they are self evident and necessarily true, although self-evidence is relative to our past knowledge .... But you may logically doubt the truth of that past knowledge thus the so called axiom is no longer self-evident. Euclidean system was supposed for centuries to be based on self-evident axioms, being indubitable ... But such supposition is now mistaken. The appearance of Non-Euclidean geometries made possible other geometries based on axioms different from Euclid's thus we reach different theorems"[20].

Criticism

This positivistic view of mathematical statements may be criticized on the following lines. First, if we agree that all statements of pure mathematics are analytic and tautologies, this does not solve the problem which empiricism faced, namely, explaining mathematical statements empirically, because it has still to explain necessity and certainty in those analytic statements. Take the typical analytical statement 'A is A'; its certainty is due to the principle of non-contradiction. This states that you cannot ascribe a predicate and its negation at the same time to a given subject. Since this principle is the ground of certainty in analytical statements, then how can we explain certainty and necessity of this principle itself? It cannot be said that the principle is itself analytic because impossibility is not involved in the being of a predicate and its negation. If we consider the principle of non-contradiction synthetic or informative, we are again required to explain its necessity. For to say that the principle is synthetic is to deny the distinction between the principle and empirical statements. On the other hand, if we admit that the principle is *a priori* not empirical we are rejecting the general grounds of empiricism. In other words, is the statement 'A is A', being certain and analytic, identical with 'it is necessary or not necessary A is A' or 'A is in fact A'? If the former then the principle is synthetic because necessity or impossibility are not involved in the concept of A; if the latter then the principle is not necessary.

Secondly, we may say that statements of applied mathematics are not absolutely necessary but have restricted necessity. For instance, axioms of Euclidean geometry are not absolutely true to any space but only true to space as plain surface; thus this geometry involves an empirical element. In consequence, it is possible that there may be other geometries different from Euclid's. But this does not deny the necessity of Euclidean axioms provided that space is a plain surface. Thus Euclidean axioms are hypothetical statements, the antecedent of which is that space is plain surface. Such statements are unempirical and thus as necessarily true as those of pure mathematics. But the former differ from the latter in that they are not analytic because the consequent is not implied in the antecedent. For that the angles of a triangle are equal to two right ones is not part of the meaning of space as plain surface. They are necessary synthetic statements.

Therefore we have to reject empiricism owing to its failure to explain the necessity of formal statements, in favour of rationalism in this respect. Further, the empiricistic dictum that sense experience is the sole source of human knowledge is not itself a logical truth, not is it itself derived from experience. It remains that this dictum is obtained *a priori*; if true, then empiricism admits *a priori* knowledge; and if it is empirical and *a priori* then it is probable. This implies that rationalism is probably true for empiricism [???].

Empiricism and Meaning of Statements

Empiricism does not only maintain that experience is the source of all knowledge, but maintains also that experience is the ground on which the meaning of statements is based. Empirically unverified statements, for empiricism, are logically meaningless, neither true nor false. That is one of Logical positivism's principal contentions.

Let us discuss this point. We have before us three theories in this context. First, we have the theory which maintains that any word having no empirical reference is without meaning, "when I am told", an eminent logical positivist writes, "that you do not understand a certain statement, this means that you cannot verify it in order to know whether it is true or false. If you tell me that there is a ponsh in this box, I understand nothing because you cannot have an image of a ponsh when you look into the box". That is to say the word 'ponsh' has no meaning because it applies to nothing in experience.

This situation depends on the view that sense is the sole source of forming concepts. If we have a statement every word in which has empirical import, it has then a meaning denoted by the possibility of forming images or concepts of each word. The following statements: 'John is a living creature', 'John is not Peter', there are bodies' are meaningful because each term in them has empirical application. Thus the statement 'there can be life without body' has a meaning because we can form a complex concept of its terms, though such concept is not in fact to be found in experience.

The second theory to explain the relation of meaning to experience states that experience makes a difference as the truth or falsehood of the statement concerned. The statement 'there can be life without body' is meaningless on this theory because the complex involved in the statement cannot be empirically tested because experience is indifferent as to disembodied life: it is not found in experience nor does experience deny it.

The third theory does not merely state that each word in a statement must have an empirical import to have a meaning, or that experience must make a difference as to the truth or falsehood of the statement. The theory states also that the statement in question must be capable of being verified. That is, unverified statements are meaningless, thus its meaning is constituted by its being verified empirically.

In consequence, a number of statements considered meaningful, if the above account is correct, are rendered by positivists meaningless. For example, 'rain has fallen in places not seen by us, has meaning on our account because its terms have empirical import and because our notion of experience makes difference as to the truth or falsity of the statement. But the statement is meaningless on the third account because it is not possible to be verified empirically, because any rain to the seen would not verify the statement. To this third account we now turn to comment.

We cannot accept the positivistic identification between the meaning and method of verifying a statement for following reasons. First, such identification is contradiction in terms, because to say of a statement that it is subject of verification or falsification is to say that it may be true or false, and *a fortiori* that it has meaning. And this involves that the meaning of a statement is not derived from its verifiability, but the latter presupposes its meaningfulness.

Secondly, there can be statements which are not only meaningful but we also believe in their truth, and yet they are empirically unverifiable; for instance, 'however wide human experience is, there can be things in nature that cannot be subject to our experience', or there can be rain falling not seen by anybody'. These statements and the like are intelligible and true although they cannot be empirically tested.

Thirdly, we may like to know what is meant by experience by virtue of which verification is possible. Is it meant to be my private experience or anyone else? If it is meant to be my own experience, this means that the statement which expresses a fact beyond my own experience has no meaning, e.g., 'there were men who lived before I was born'; but this certainly is meaningful. Further, if by experience is meant that of other people, this is inconsistent with positivistic principles because experiences of other minds lie beyond my own, but they are known to me inductively. Thus, any such statement is meaningful. For example, the belief in causality as involving necessary relation between cause and effect has meaning though it is not immediately verified by me but it is inductively verified.

Fourthly, we may ask again, whether the criterion of the meaning of a statement is its actual verification or its verifiability. If we assumed the former, then what cannot be actually verified is meaningless, for logical positivism, even if the statement is concerned with nature. The statement 'the other side of the moon is full of hills and valleys' is not actually verified because this other side is not seen by anyone on the earth and so no one is able to verify its truth. However, it is false to say with the positivists that such statement is meaningless.

Science often provides propositions to be examined even before we possess the crucial experiment which testifies their truth. And scientific activity in testing hypotheses would be frivolous if scientific hypotheses were meaningless.

On the other hand, suppose we assume that the positivists actually claim that the meaning of a statement is its verifiability in principle, in the cases in which actual verification is empirically impossible but still logically possible. We must now examine this claim, how do we know that a statement is verifiable? If we do know this in a way different from sense experience, then positivists admit a sort of knowledge independent of experience which is inconsistent with its principles. And if they identify verifiability with actual verification, they consider many statements meaningless though they are concerned with nature and are intelligible.

In fact we need understand a criterion of the meaning of a statement before we test its truth or falsity. Truth or falsity presupposes one image comprising the concepts of the terms and the relations among them in a statement. If we can grasp such complex image, we can get its meaning.

Has knowledge Necessarily A Beginning?

If human knowledge is established such that certain items are derived from others either by deduction or induction, then it must have a beginning with certain premises un-derived in any way. For otherwise we fall in an infinite regress, and thus knowledge becomes impossible.

Reichenbach's Position

Reichenbach claims the possibility of knowledge without any beginning, and argues (a) that human knowledge is all probable, (b) that probable knowledge can be explained in terms of probability theory, and (c) that the theory of probability he adheres to is frequency theory, and that the proportion of the frequency of past events is constant and regular. In consequence, any probability involves a certain frequency, the proportion of which can be determined by means of other frequency probabilities, without beginning. Lord Russell illustrates Reichenbach's theory by the example of the chance that an English man of sixty will die within a year. "The first stage is straightforward: Having accepted the records as accurate, we divide the number of dead people within the last year by the total number. But we now remember that each item in the statistics may get some set of similar statistics which has been carefully scrutinized, and discover what percentage of mistakes it contained. Then we remember that those who thought they recognised a mistake may have been mistaken, and we set to work to get statistics of mistakes about mistakes. At some stage in this regress we must stop; wherever we stop, we must conventionally assign a "weight" which will presumably be either certainty or the probability which we guess would have resulted from carrying our regress one stage further"[21].

Russell's Objection

Russell objects to this point of Reichenbach by saying that this infinite regress makes the value of probability determined in the first stage of the regress almost zero. For we can say the probability that an (a) will be a (b) is m_1/n_1; at the level, we assign to this statement a probability m_2/n_2, by making it one of some series of similar statements; at the third level, we assign a probability m_3/n_3 to the statement that there is a probability m_2/n_2 in favour of our first probability m_1/n_1 and so we go on for ever. If this endless regress could be carried out, the ultimate probability in favour of the rightness of our initial estimate m_1 / n_1 would be an infinite product: $m_2/n_2 . m_3/n_3 . m_4/n_4$ .... which may be expected to be zero. It would seem that in choosing the estimate which is most probable at the first level we are almost sure to be wrong[22].

Discussion

But Russell's objection may be retorted by saying that any estimation we impose on endless regress which may be mistaken admits of two alternatives: the mistake may arise when we realise that the proportion of mistakes in statistics is greater than that which we found in the list discovering mistakes in this statistics, or the mistake arises when we realise that the former proportion is lesser than the latter. For example, so we may suppose that the value of the probability of the death rate among Englishmen over sixty is 1/2, on the ground of the frequency of death rate in statistical records.

Now if we look back into these records and found that the rate of mistakes in such records is 1/10, this means that the value 1/2 has the chance that it may be mistaken with the probability value 1/10. Thus the possibility of mistake involves two equal probabilities, i.e., either that the first value is really over 1/2, or that the second is really less, not that the value is 1/2 x 1/10.

We believe that Reichenbach is mistaken in dispensing with the absolute beginning of knowledge by recourse to endless regress. For no knowledge is possible without real starting point.

For instance, the probability which determines our knowledge that Englishmen over sixty die cannot be interpreted except in terms of probability theory with all the axioms and postulates connected with it. Thus, in applying such theory, we have to assume prior knowledge of those axioms, and these constitute our starting point. And those axioms cannot be applied, as we having already shown, except on the basis of indefinite knowledge. Therefore there can be probable knowledge without prior knowledge.

As to the beginnings of knowledge, we may assume two kinds of knowledge: one presupposed by the axioms of theory of probability, the other is that of the nature of sensible experience regardless of its contents. When we see clouds in the sky for example, then clouds make the object of our seeing, but our awareness of seeing is a primary knowledge and not inferred. Now, we may ask whether such primary knowledge is certain or not. It is not necessarily certain but may be probable.

Primary probable knowledge applies to two fields. First, it applies to sensible experience. Usually I am certain about what I experience, but it may happen that I am doubtful about what I see or hear when the object is dull or faint or distant in my perceptual field; in this case I get probable knowledge. The second field of primary probable knowledge is that of primary propositions in which the relation of subject to predicate is immediate without a middle term. Such propositions are the ground of all syllogistic inferences, and can themselves be reached by direct awareness. Such awareness may get the utmost degree of certainty, and may gain lesser degree of credibility. In consequence, since those propositions may have probability values we may increase their values by virtue of probability theory.

Notes:

[19]Hume claimed the a priori character of formal statements before Logical Positivists.

[20]Zaki Naguib Mahmoud. Positivistic Logic, P. 324, Cario. 1951

[21](1) Russell, Human Knowledge, P. 433.

[22]Ibid, p. 434